EAE 127 – Applied Aerodynamics
Homework 6

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Section: EAE 127-001 (FQ25)

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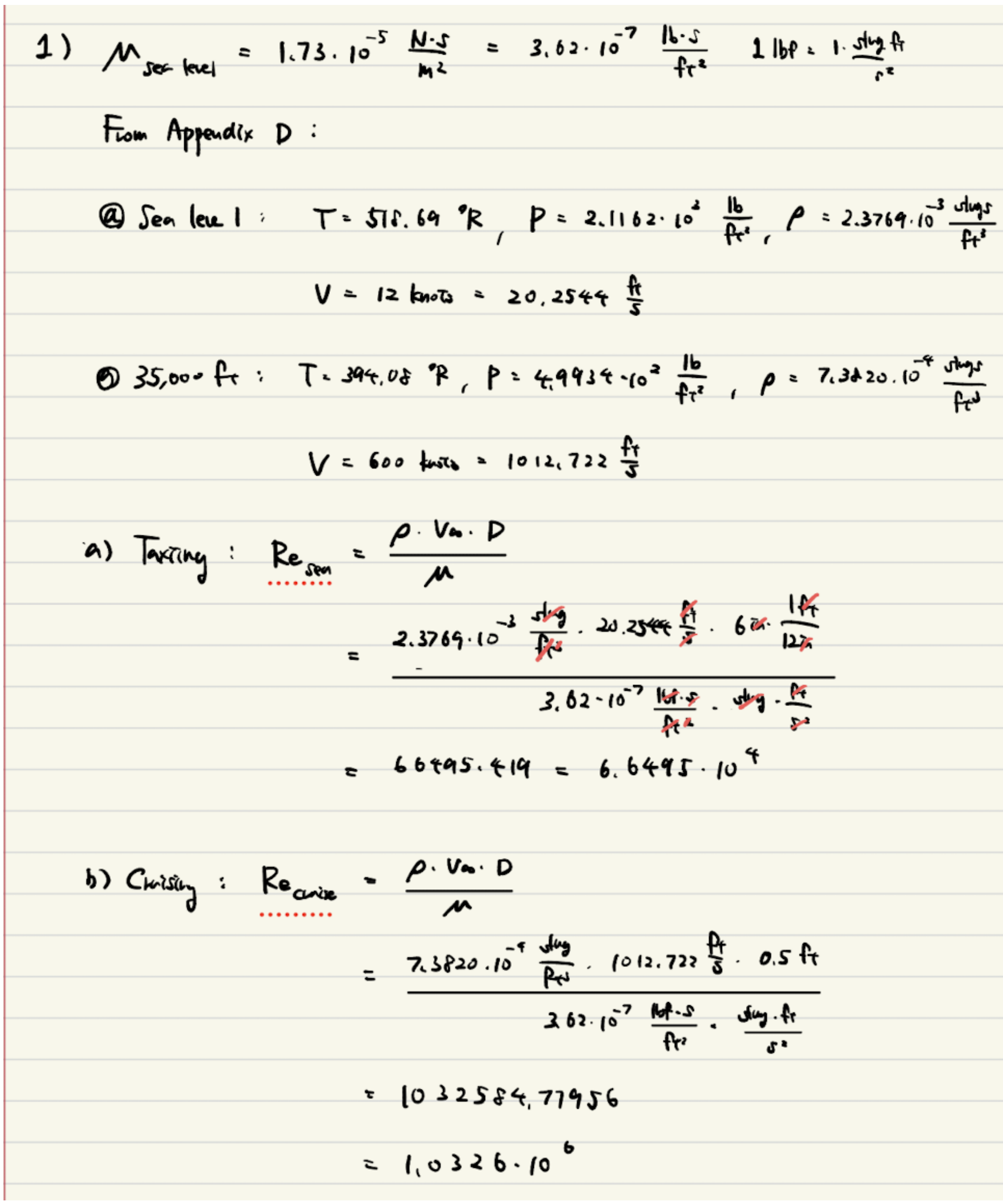
1. **Boundary Layers for Different Flight Regimes**
   (a) Compute the Reynolds number for the T-38 antenna at:
      - Sea-level taxiing
      - Cruise at altitude

   Use the antenna chord of 6 in and assume constant sea-level viscosity.
      - Appropriateness of laminar vs turbulent BL assumptions
      - Validity of incompressible-flow assumption for each regime



1) $\mu_{sea\ level} = 1.73 \cdot 10^{-5} \frac{N \cdot s}{m^2} = 3.62 \cdot 10^{-7} \frac{lb \cdot s}{ft^2}$ $\quad 1\ lbf = 1 \cdot \frac{slug \cdot ft}{s^2}$

From Appendix D:

@ Sea level: $T = 518.69\ °R$, $P = 2.1162 \cdot 10^{3} \frac{lb}{ft^2}$, $\rho = 2.3769 \cdot 10^{-3} \frac{slugs}{ft^3}$

$V = 12\ knots = 20.2544 \frac{ft}{s}$

@ 35,000 ft: $T = 394.08\ °R$, $P = 4.9934 \cdot 10^{2} \frac{lb}{ft^2}$, $\rho = 7.3820 \cdot 10^{-4} \frac{slugs}{ft^3}$

$V = 600\ knots = 1012.722 \frac{ft}{s}$

a) Taxiing: $Re_{sea} = \frac{\rho \cdot V_\infty \cdot D}{\mu}$

$$= \frac{2.3769 \cdot 10^{-3} \frac{slug}{ft^3} \cdot 20.2544 \frac{ft}{s} \cdot 6\ in \cdot \frac{1\ ft}{12\ in}}{3.62 \cdot 10^{-7} \frac{lbf \cdot s}{ft^2} \cdot \frac{slug \cdot ft}{s^2}}$$

$= 66495.419 = 6.6495 \cdot 10^4$

b) Cruising: $Re_{cruise} = \frac{\rho \cdot V_\infty \cdot D}{\mu}$

$$= \frac{7.3820 \cdot 10^{-4} \frac{slug}{ft^3} \cdot 1012.722 \frac{ft}{s} \cdot 0.5\ ft}{3.62 \cdot 10^{-7} \frac{lbf \cdot s}{ft^2} \cdot \frac{slug \cdot ft}{s^2}}$$

$= 1032584.77956$

$= 1.0326 \cdot 10^6$

Figure 1: Calculations 1/2

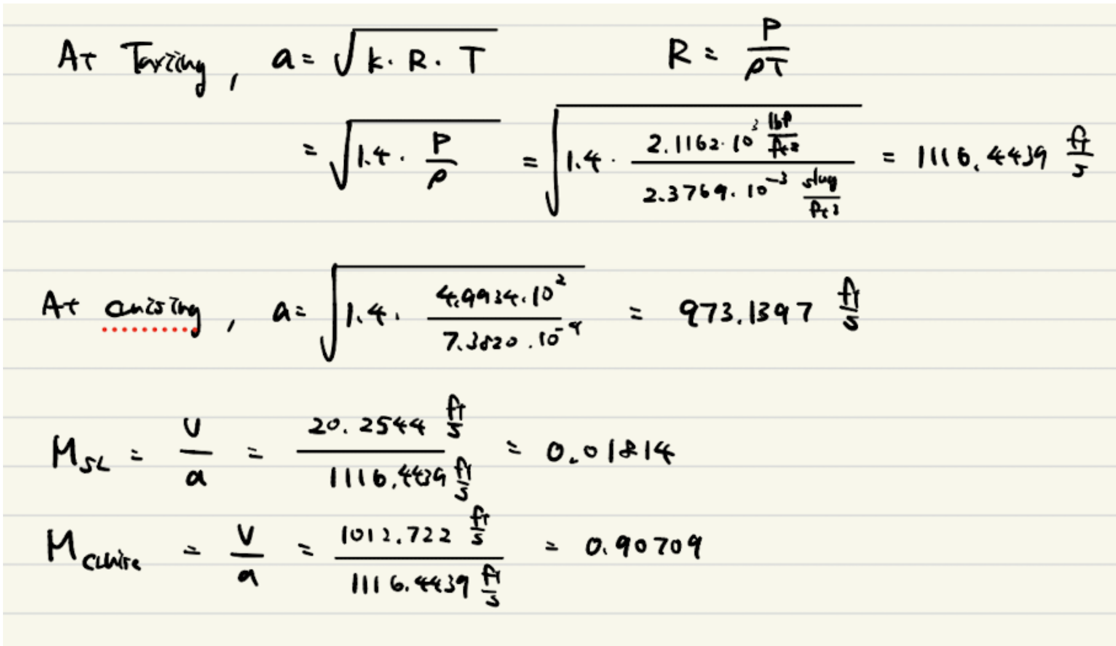


Figure 2: Calculations 2/2

Ans:

- $Re_{\text{taxi}} = 6.6495 \times 10^4$

  This is smaller than the transition Reynolds number of 5×10^5. Because the aircraft is taxiing, the velocity is relatively low, and it is safe to assume that the flow is incompressible.

- $Re_{\text{cruise}} = 1.0326 \times 10^6$

  This is well above 5×10^5, which means the cruise boundary layer is fully turbulent. However, at the altitude and speed at which the airplane is cruising, the Mach number is about 0.9, which is transonic and highly compressible. At this altitude, both the air density and viscosity vary significantly. Therefore, it is not valid to assume incompressible flow for the cruise case.

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2. **Velocity Profile for Laminar and Turbulent Boundary Layers**

   (a) Using:

$$\left.\frac{u}{u_e}\right|_{\text{lam}} = \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right), \qquad \left.\frac{u}{u_e}\right|_{\text{turb}} = \left(\frac{y}{\delta}\right)^{1/7}$$

   and the Blasius thickness formulas:

$$\delta_{\text{lam}} = \frac{5.0\,x}{\sqrt{Re_x}}, \qquad \delta_{\text{turb}} = \frac{0.16\,x}{(Re_x)^{1/7}}$$

compute the velocity profiles at $x = 6$ in.

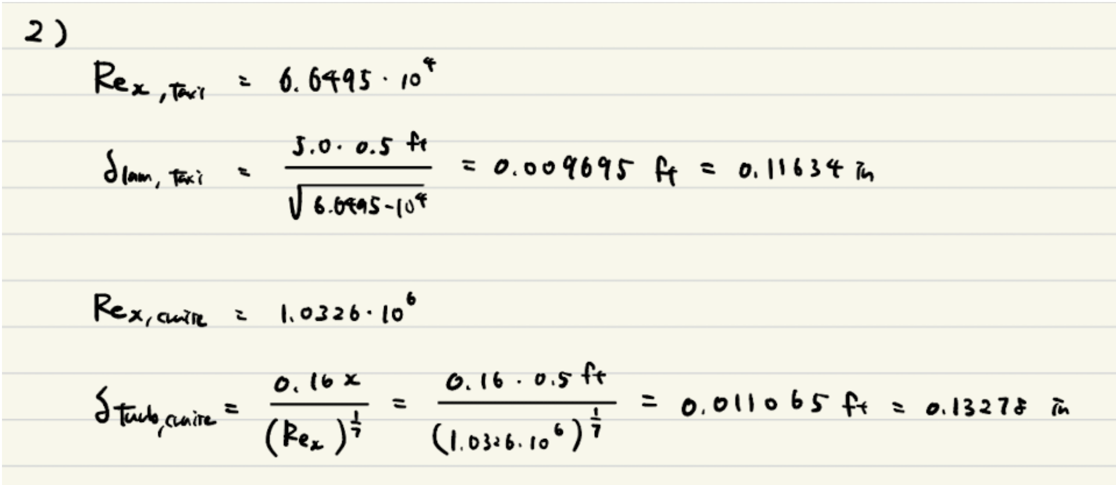

Figure 3: Calculations for δ for each case

(b) Plot:

1. Dimensional u vs. dimensional height y

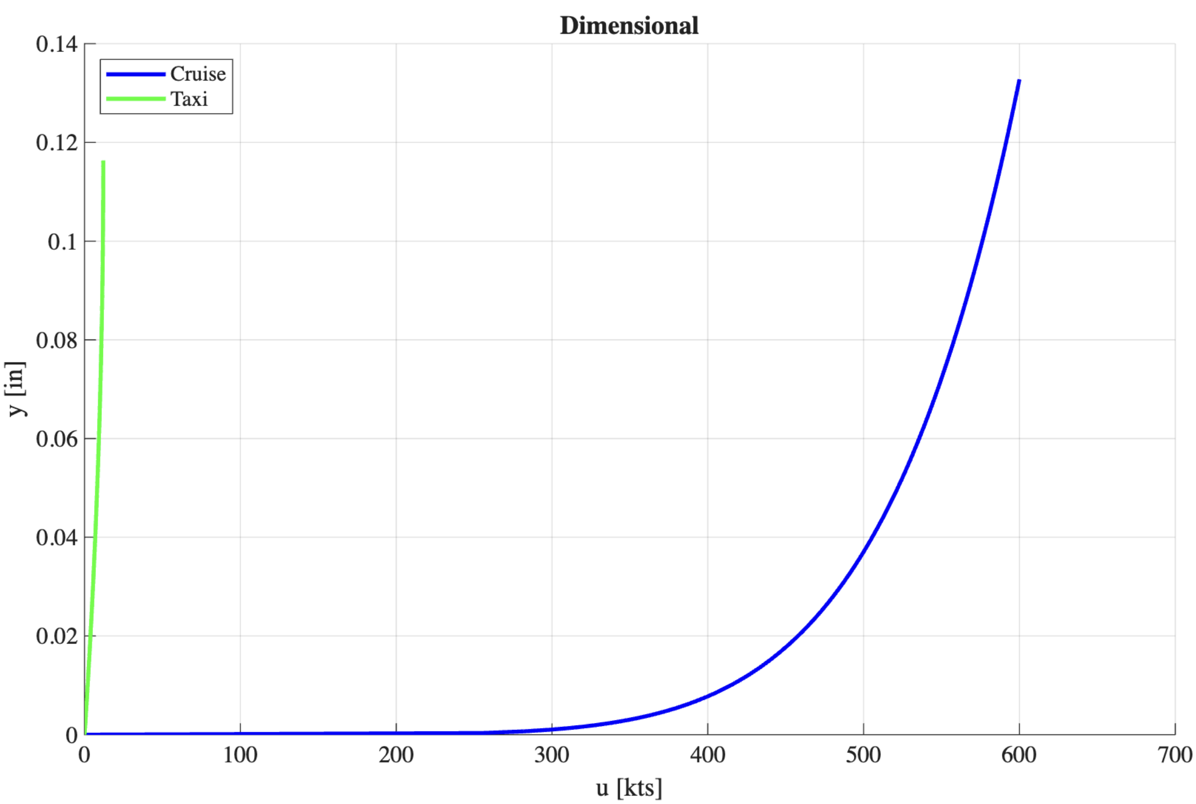

Figure 4: Dimensional graph

2. Non-dimensional u/u_e vs. y/δ

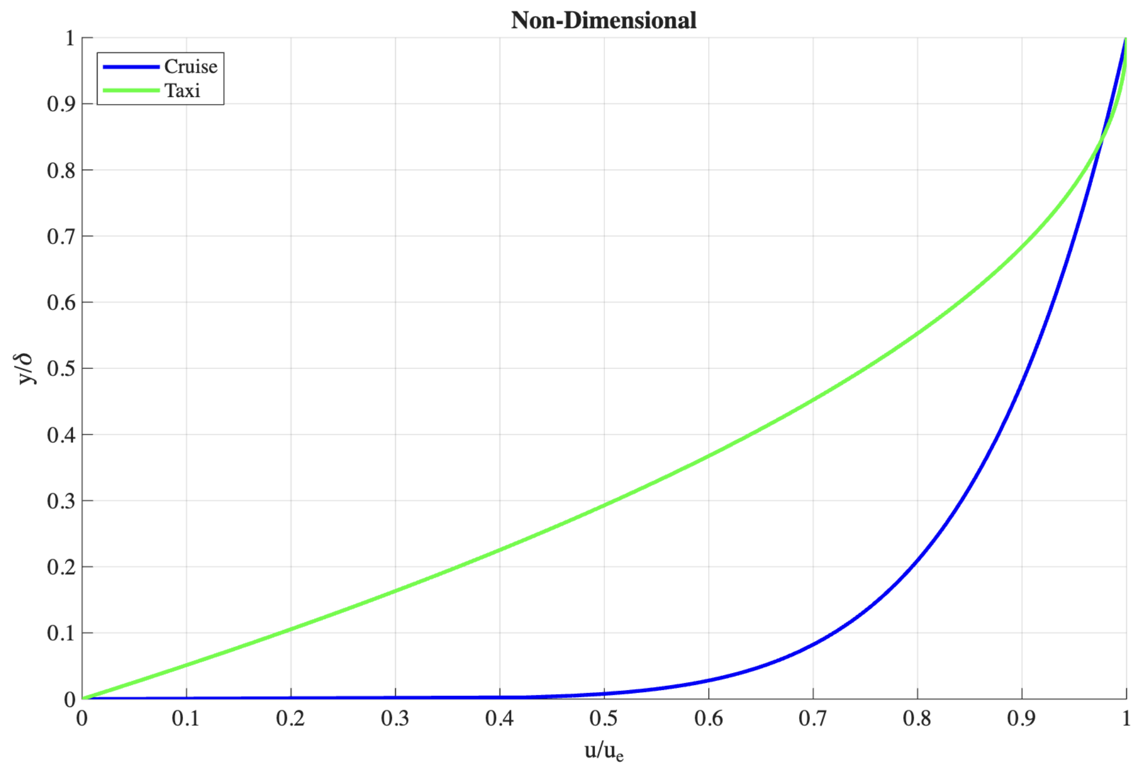

Figure 5: Non-Dimensional graph

(c) Discuss:

- Differences between laminar and turbulent profiles

Ans:

  - In the non-dimensional plot, the laminar/taxi curve rises almost linearly across the boundary layer. In the dimensional plot, the laminar curve is extremely narrow because the freestream velocity during taxi is very small.
  - In the non-dimensional plot, the turbulent boundary layer shows a steep velocity gradient near the wall, followed by a rapid rise to u/u_e = 1. In the dimensional plot, the turbulent curve extends far to the right because the cruise freestream velocity is much higher.

- Why turbulent BL resists adverse pressure gradients

Ans: By definition, an adverse pressure gradient occurs when pressure increases in the direction of the flow along the surface. This slows the fluid and encourages flow separation.

  - As shown in the dimensional graph above, the laminar boundary layer has a very narrow shape, indicating low near-wall momentum. This means that when pressure rises, the low-momentum fluid quickly stalls and leads to early flow separation.

– As seen in the non-dimensional plot, the turbulent boundary layer profile has a steeper gradient, which means stronger shear and higher momentum exist in the layer where resistance to separation is most needed.

- Why turbulent BL produces higher skin friction

Ans:

– In laminar flow, $\frac{\partial u}{\partial y}$ is small because the slope near the wall is shallow. This indicates low wall shear stress and therefore low friction drag.

– In turbulent flow, the velocity rises very steep near the wall, which means $\frac{\partial u}{\partial y}$ is large. This creates a high wall shear stress and causes higher friction drag.

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3. **Friction Drag for Different Flow Regimes**

Using flat-plate relations:

$$c_{f,\text{lam}} = \frac{1.328}{\sqrt{Re_c}}, \qquad c_{f,\text{turb}} = \frac{0.074}{(Re_c)^{1/5}}$$

(a) Compute total drag coefficient due to skin friction on the antenna:

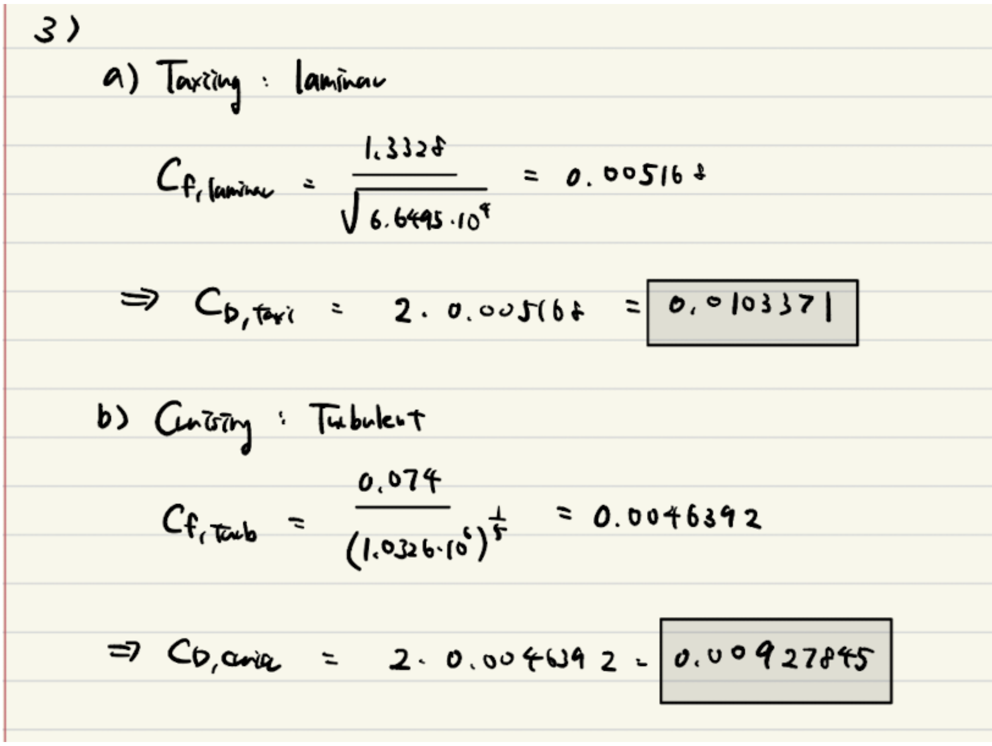

Figure 6: Calculations

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4. **Viscid–Inviscid Interaction**

   (a) Define:

   - Boundary layer thickness δ

     Ans: The boundary layer thickness $\delta(x)$ is the distance from the wall where the local velocity is essentially equal to the freestream velocity. In mathematical terms, $u(x, \delta) = 0.99\, u_e(x)$. It represents the effective thickness of viscous influence within the flow; outside the boundary layer, the flow is essentially inviscid.

   - Displacement thickness δ^*

     Ans: The displacement thickness δ^* gives rise to the concept of an effective body, as it measures how much the boundary layer displaces the inviscid streamlines outward. The boundary layer slows the fluid near the wall due to viscosity, which introduces a "missing mass flow" compared to inviscid flow, and this is equivalent to the wall being "thicker" than it really is. In other words, the displacement thickness represents how much thicker the airfoil appears to the inviscid flow because of viscosity.

   - Momentum thickness θ

Ans: The momentum thickness θ is an index proportional to the loss in momentum flow due to the presence of the boundary layer and the shear within it. It can be interpreted as the height of a hypothetical streamtube that would carry the "missing" momentum at freestream conditions. Because it directly relates to wall shear stress, it also relates to drag. Thus, the momentum thickness represents the momentum deficit caused by the boundary layer.

(b) Provide an outline of the iterative algorithm:

1. **Start with original geometry.**
   - Input: airfoil/panel geometry.
2. **Compute inviscid flow.**
   - Run potential flow to obtain $u_e(x)$.
3. **Compute boundary layer properties.**
   - Using $u_e(x)$, solve BL equations or correlations to obtain $\delta(x)$, $\delta^*(x)$, $\theta(x)$, $C_f(x)$.
4. **Modify geometry using displacement thickness.**
   - For each surface point, $y_{new}(x) = y_{original}(x) + \delta^*(x)$.
5. **Recompute inviscid flow on the displaced geometry.**
   - Run panel method on the displaced geometry, which creates a new surface velocity distribution $u_{e,new}(x)$.
6. **Check for convergence.**
   If $u_{e,new}(x)$ - $u_e(x)$ ¡ tolerance, it converged. If else, it repeats back to 3-6.

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5. **XFOIL Analysis: NACA 23012**

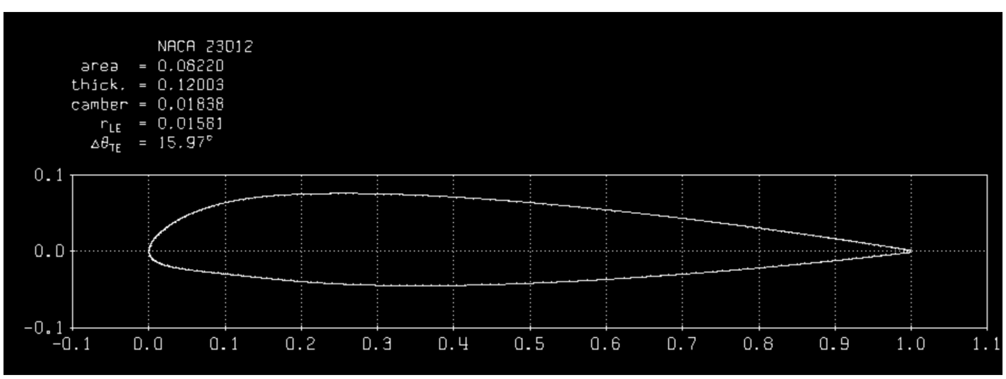

Figure 7: NACA 23012 airfoil geometry

(a) Run XFOIL at:

$$Re = 1.37 \times 10^5, \qquad M = 0.01, \qquad \alpha = 0^\circ$$

Record:

- Inviscid NACA 23012 C_L, C_D

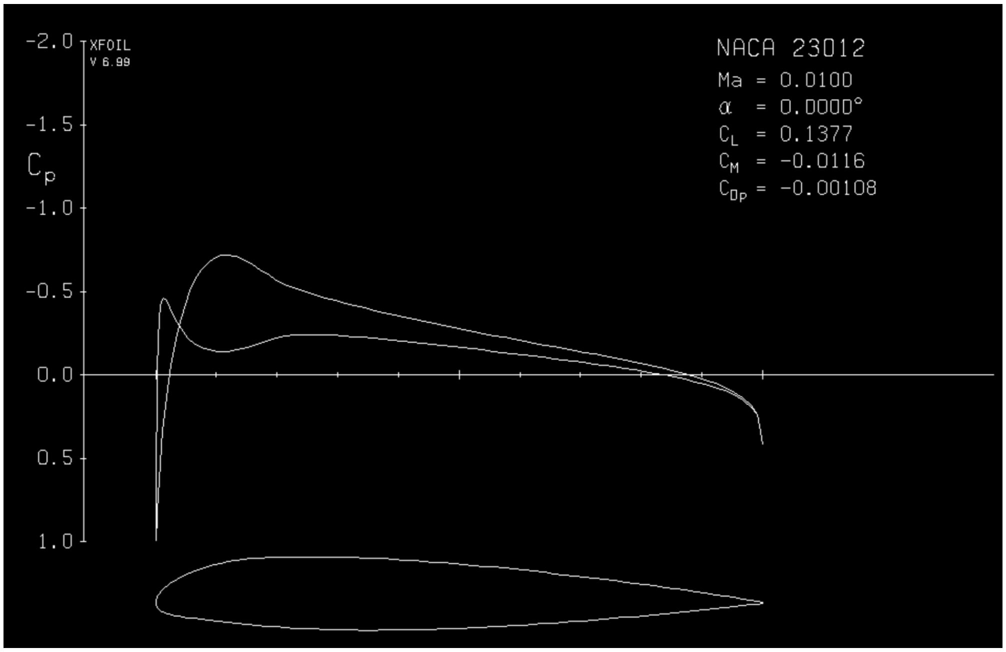

Figure 8: Inviscid C_P vs. x/C

- Viscid NACA 23012 C_L, C_D

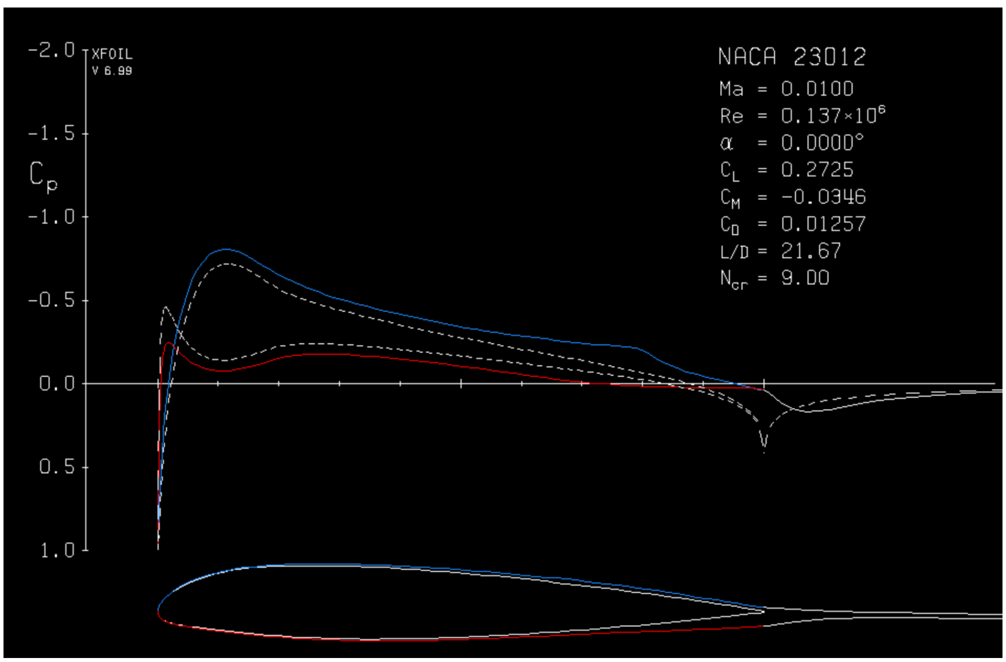

Figure 9: Viscid C_P vs. x/C

- Inviscid modified NACA 23012 C_L, C_D

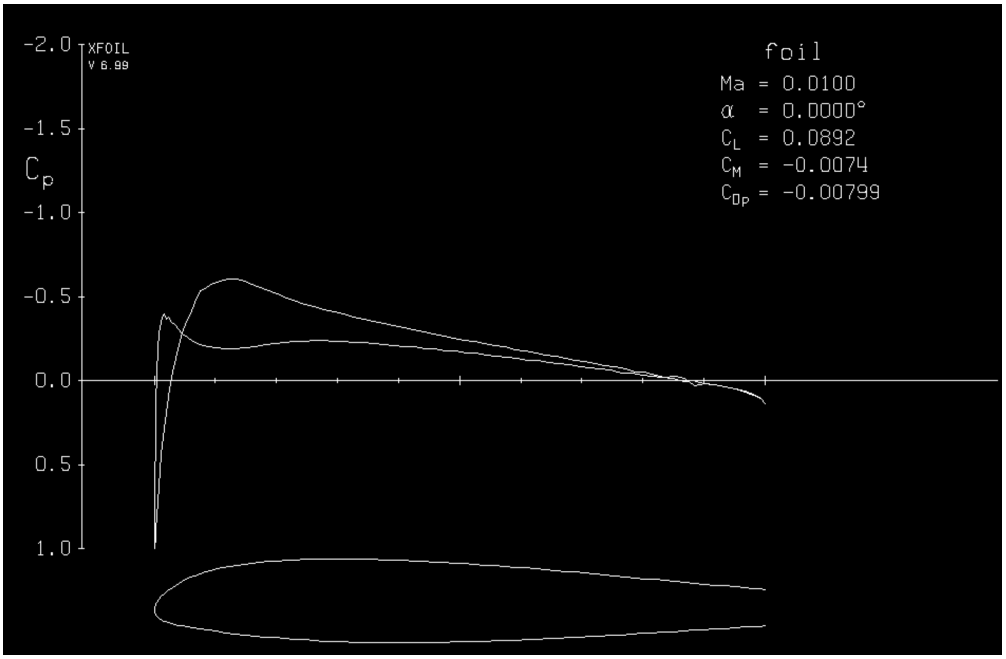

Figure 10: Inviscid modified NACA 23012 C_P vs. x/C

Coefficients	NACA 23012 (inviscid)	NACA 23012 (viscid)	NACA 23012-3-modified (inviscid)
C_L	0.1377	0.2725	0.0892
C_D	-0.00108	0.01257	

Table 1: Lift and drag coefficients

Ans: According to the MATLAB code, we get the cumulative drag coefficient of 0.0105, which is close but not exactly the same as the viscous result, which can be a result of XFoil's limited resolution.

(b) Explain viscous decambering and how boundary layer growth reduces effective camber and lift.

Ans: The modified inviscid C_L is lower than both the viscid and inviscid NACA 23012 results. This reflects the effect of viscous decambering, in which the boundary layer effectively thickens the airfoil. As seen in the figure above, the trailing edge of the modified airfoil is thicker than that of the original airfoil. Because adding δ^* to the surface shape deliberately reduces curvature, the modified airfoil generates less circulation and therefore lower lift, effectively reducing its geometric camber.

MATLAB Code

```
clear; clc;

x = 0.5;  % ft

rho_taxi = 2.3769e-3;
mu_taxi  = 3.62e-7;
U_taxi   = 20.2544;

rho_cruise = 7.3820e-4;
mu_cruise  = 3.62e-7;
U_cruise   = 1012.722;

Rex_taxi   = rho_taxi   * U_taxi   * x / mu_taxi;
Rex_cruise = rho_cruise * U_cruise * x / mu_cruise;

% Taxi: laminar
delta_taxi   = 5*x / sqrt(Rex_taxi);

% Cruise: Turbulent
delta_cruise = 0.16*x / (Rex_cruise^(1/7));

Ny = 400;
y_taxi   = linspace(0, delta_taxi,   Ny);
y_cruise = linspace(0, delta_cruise, Ny);

% Non-dimensional
u_over_ue_taxi   = 2*(y_taxi/delta_taxi) - (y_taxi/delta_taxi).^2;      % laminar
u_over_ue_cruise = (y_cruise/delta_cruise).^(1/7);                      % turbulent

% Dimensional
u_taxi_ft   = u_over_ue_taxi   * U_taxi;
u_cruise_ft = u_over_ue_cruise * U_cruise;

% Convert to knots
ftps_to_kts = 1/1.68781;
u_taxi_kts   = u_taxi_ft   * ftps_to_kts;
u_cruise_kts = u_cruise_ft * ftps_to_kts;

figure('Color','w'); hold on;

plot(u_cruise_kts, y_cruise*12, 'b', 'LineWidth', 2);  % Cruise (blue)
plot(u_taxi_kts,   y_taxi*12,   'g', 'LineWidth', 2);  % Taxi (green)

xlabel('u [kts]');
ylabel('y [in]');
title('Dimensional');
legend('Cruise','Taxi','Location','northwest');
grid on;
set(gca,'FontName','Times','FontSize',12);
```

```
figure('Color','w'); hold on;

plot(u_over_ue_cruise, y_cruise/delta_cruise, 'b', 'LineWidth', 2);
plot(u_over_ue_taxi,   y_taxi/delta_taxi,     'g', 'LineWidth', 2);

xlabel('u/u_e');
ylabel('y/\delta');
title('Non-Dimensional');
legend('Cruise','Taxi','Location','northwest');
grid on;
set(gca,'FontName','Times','FontSize',12);
```

Listing 1: MATLAB code for Question 2