EAE 127 – Applied Aerodynamics
Homework 7: Airfoil Performance

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Student: Kaitaro Hori
Student ID: 920584376
Section: EAE 127-001 (FQ25)

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1. **Course 1 Airfoil Selection Process**

   (a) **Design Conditions**

   Given:

   - Wing area: $S = 235\,\text{ft}^2$
   - Root chord length: $c = 8.48\,\text{ft}$
   - Wing span: $b = 45\,\text{ft}$
   - Cruise speed: $V_{\text{cruise}} = 400\,\text{mph}$
   - Geometric cruise altitude: $h = 22\,000\,\text{ft}$
   - Loaded weight: $W = 12\,000\,\text{lb}$
   - Dynamic viscosity at h: $\mu = 3.25 \times 10^{-7}\ \text{slug}/(\text{ft} \cdot \text{s})$

At $h = 22{,}000$ ft, according to Appendix E:

$$\begin{cases} T = 440.32\ {}^\circ R \\ P = 8.9459 \cdot 10^{2}\ \dfrac{\text{lbf}}{\text{ft}^2} \\ \rho = 1.1836 \cdot 10^{-3}\ \dfrac{\text{slugs}}{\text{ft}^3} \end{cases}$$

$$V = 400\ \text{mph} \cdot \frac{1.4667\ \text{ft/s}}{1\ \text{mph}} = 586.66\ \text{ft/s}$$

$$\begin{aligned} Re &= \frac{\rho V_{\text{cruise}} \cdot c}{\mu} \\ &= \frac{1.1836 \cdot 10^{-3}\ \frac{\text{slug}}{\cancel{\text{ft}^3}} \cdot 586.66\ \frac{\cancel{\text{ft}}}{\cancel{\text{s}}} \cdot 8.48\ \cancel{\text{ft}}}{3.25 \cdot 10^{-7}\ \frac{\text{slug}}{\cancel{\text{ft} \cdot \text{s}}}} \\ &= \boxed{1.8117 \cdot 10^{7}} \end{aligned}$$

$$a = \sqrt{k \cdot R \cdot T} \quad \text{Assuming the air is ideal condition, } P = \rho R T$$

$$\begin{aligned} &= \sqrt{k \cdot \frac{P}{\rho T} \cdot T} \\ &= \sqrt{1.4 \cdot \frac{8.9459 \cdot 10^{2}}{1.1836 \cdot 10^{-3}}} = 1028.664\ \frac{\text{ft}}{\text{s}} \Rightarrow M = \frac{V}{a} = \frac{586.66}{1028.664} \\ &\qquad\qquad = 0.5703 \end{aligned}$$

Subsonic

$$AR = \frac{b}{c} = \frac{45\ \text{ft}}{8.48\ \text{ft}} = 5.3066$$

$$\begin{aligned} C_{L,\text{cruise}} &= \frac{L}{q \cdot S} = \frac{12{,}000\ \text{lbf}}{\frac{1}{2} \cdot 1.1836 \cdot 10^{-3}\ \frac{\text{slug}}{\cancel{\text{ft}^3}} \cdot 586.66^2\ \frac{\cancel{\text{ft}^2}}{\text{s}^2} \cdot 235\ \cancel{\text{ft}^2}} \\ &= 0.2507 \end{aligned}$$

$$C_{L,2g} = 2 \cdot C_{L,\text{cruise}} = 0.501413$$

Figure 1: Calculations for Re number, $C_{L,\text{cruise}}$, and $C_{L,2g}$.

Ans:

- $\mathrm{Re} = 1.8117 \times 10^7$
- $C_{L,\text{cruise}} = 0.2507$
- $C_{L,2g} = 0.5014$

(b) **Airfoil Analysis**

Consider the following airfoils:

- NACA 0012
- NACA 23012
- P-51D Root

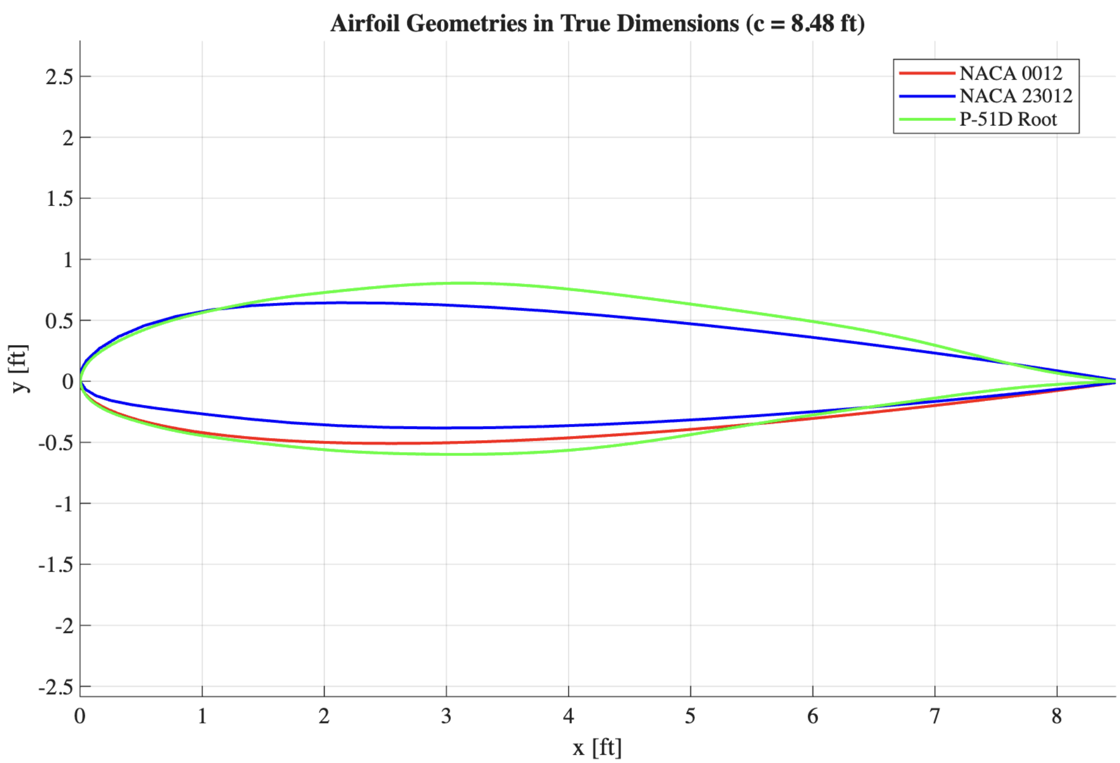

Figure 2: NACA 0012, NACA 23012, and P-51D airfoil geometries in true dimensions.

Ans: The P-51D airfoil exhibits the greatest overall thickness, and its maximum thickness location is the furthest aft among the three airfoils. This aft-shifted thickness distribution is a very characteristic of laminar airfoils designed to delay transition and reduce drag.

XFOIL polar data:

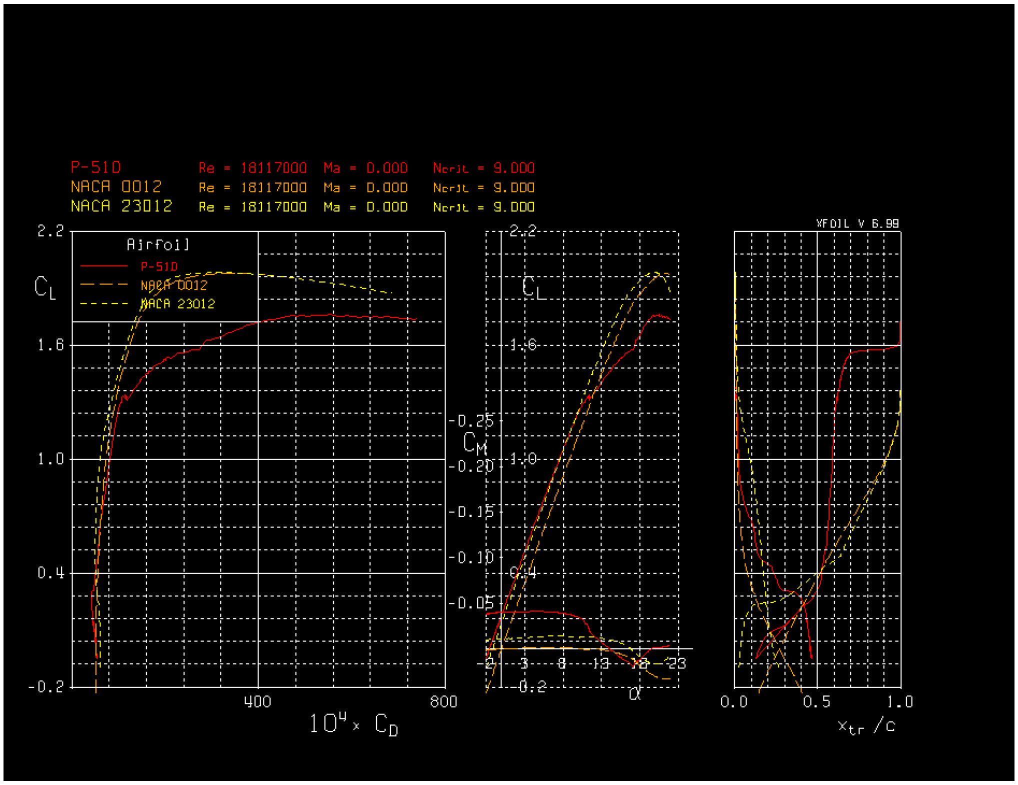

Figure 3: Xfoil results of the airfoils in one plot.

1. **Viscous lift curves** (c_l vs. α)

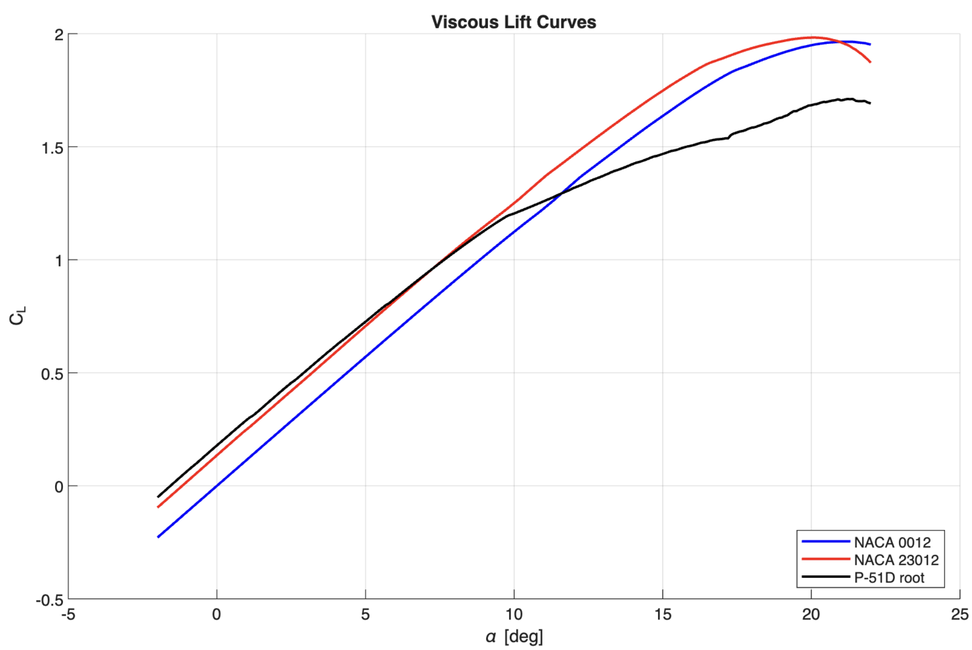

Figure 4: Viscous lift curves c_l vs. angle of attack α for all airfoils.

2. **Viscous drag polars** (c_l vs. c_d), two views:
   (a) Limited axes: $-2 < c_l < 2$, $0 < c_d < 0.025$

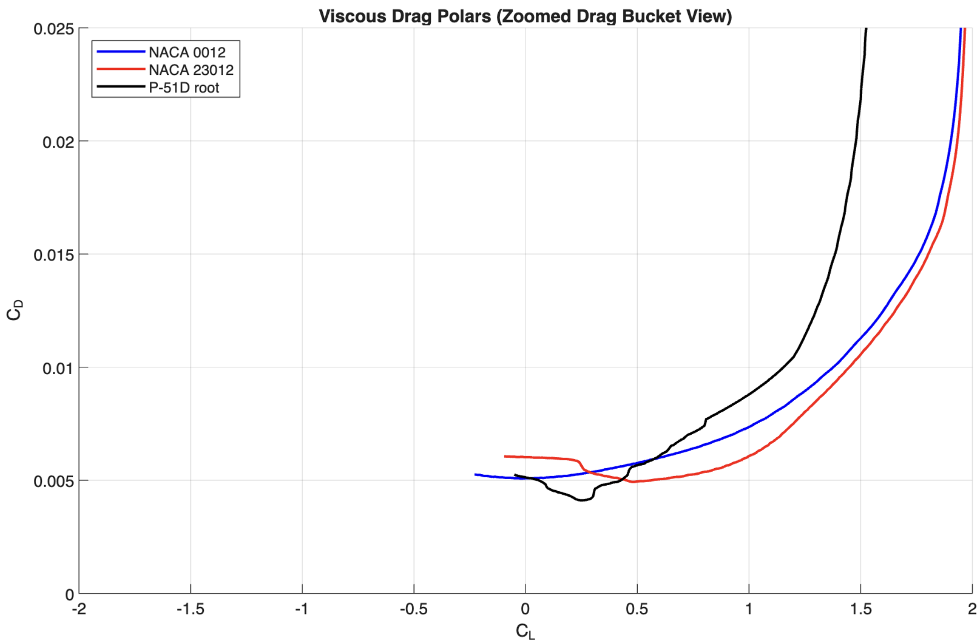

Figure 5: Viscous drag polars (low- angle-of-attack trends)

   (b) Full c_d range to show high-α behavior

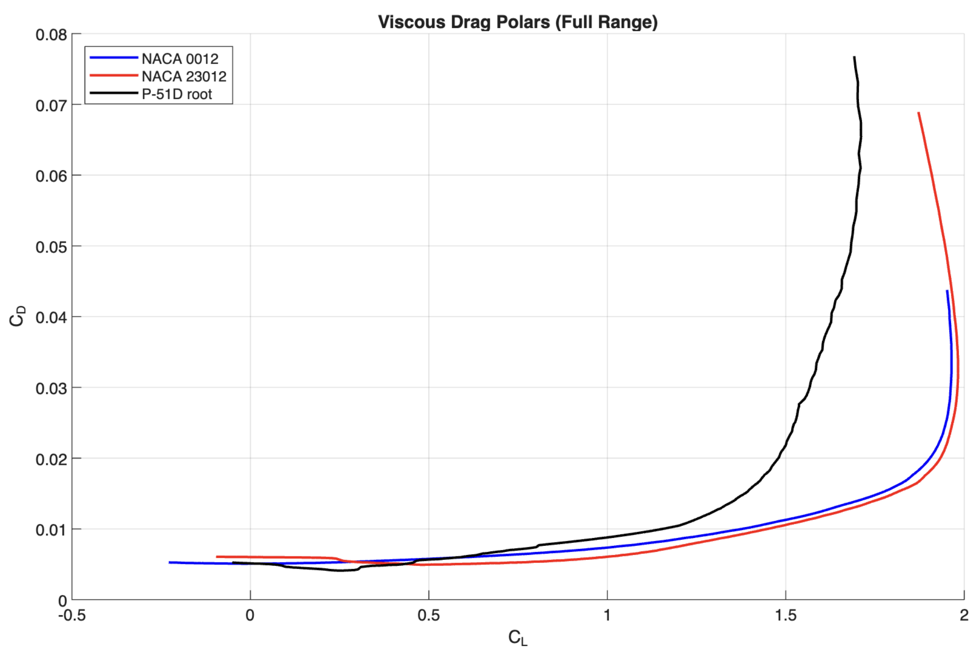

Figure 6: Viscous drag polars (high AoA behavior).

3. **Lift-to-drag ratio** (L/D vs. α)

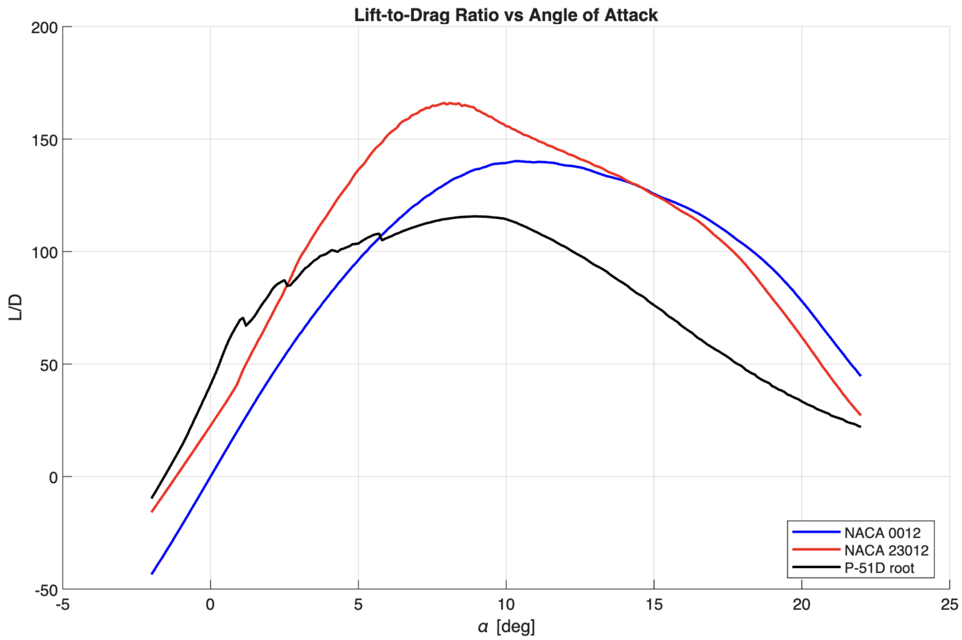

Figure 7: Lift-to-drag ratio L/D vs. angle of attack for all airfoils.

Ans:

(a) The NACA 23012 airfoil exhibits the highest $C_{L,\max}$ of the three geometries. The symmetric NACA 0012 produces $C_L = 0$ at $\alpha = 0°$, consistent with its zero-camber design. The P-51D airfoil shows a noticeable change in lift-curve slope near $\alpha \approx 10°$.

(b) The P-51D airfoil reaches its minimum C_D at the lowest C_L, with a small drag bucket from 0° to 0.5°. The bucket occurs as its aft camber sustains laminar flow and a favorable pressure gradient over the chord. The NACA 23012 airfoil has a wider bucket (about 0.25° to 1°), caused by extended laminar flow due to its camber and moderate thickness. The NACA 0012 does not exhibit a bucket since its symmetric shape causes earlier transition and no sustained laminar pressure recovery.

(c) The drag polars for the NACA 23012 and NACA 0012 show similar steep curvature as lift increases. In contrast, the P-51D airfoil exhibits a smoother and more gradual curvature, consistent with a wider laminar recovery region.

(d) The NACA 23012 airfoil demonstrates the highest lift-to-drag ratio overall, producing a distinctly triangular L/D curve. The NACA 0012 exhibits a lower, more rounded L/D profile. The P-51D airfoil has a rougher curvature and remains relatively flat between $\alpha = 0°$ and 10°, consistent with its gradual laminar pressure recovery.

(e) **Performance Analysis for P-51D Airfoil**

- Using XFOIL, determine the angle of attack corresponding to:
  - Level-flight condition ($C_{L,\text{cruise}}$)
  - $2g$ turn condition ($C_{L,2g}$)
  - Stall
- Use the `CL` command in XFOIL to find the angles and record c_p distributions.
- Extract the three C_p vs. x/c plots and overlay them in a single figure.

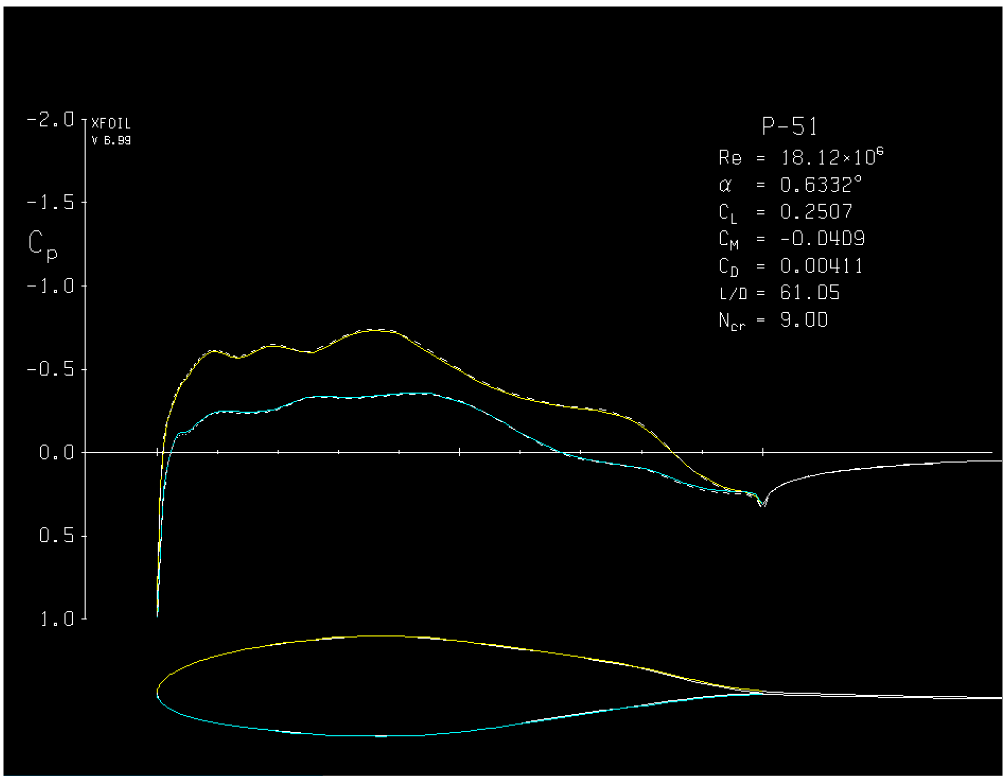

Figure 8: C_p distributions for level flight on Xfoil

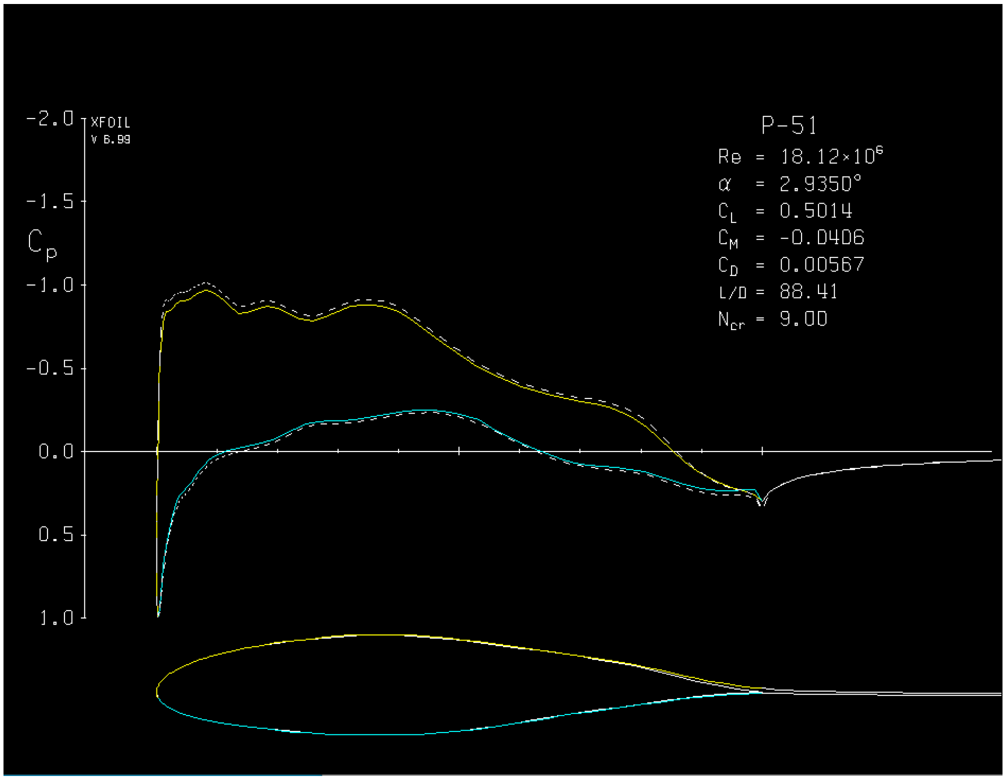

Figure 9: C_p distributions for 2g flight on Xfoil

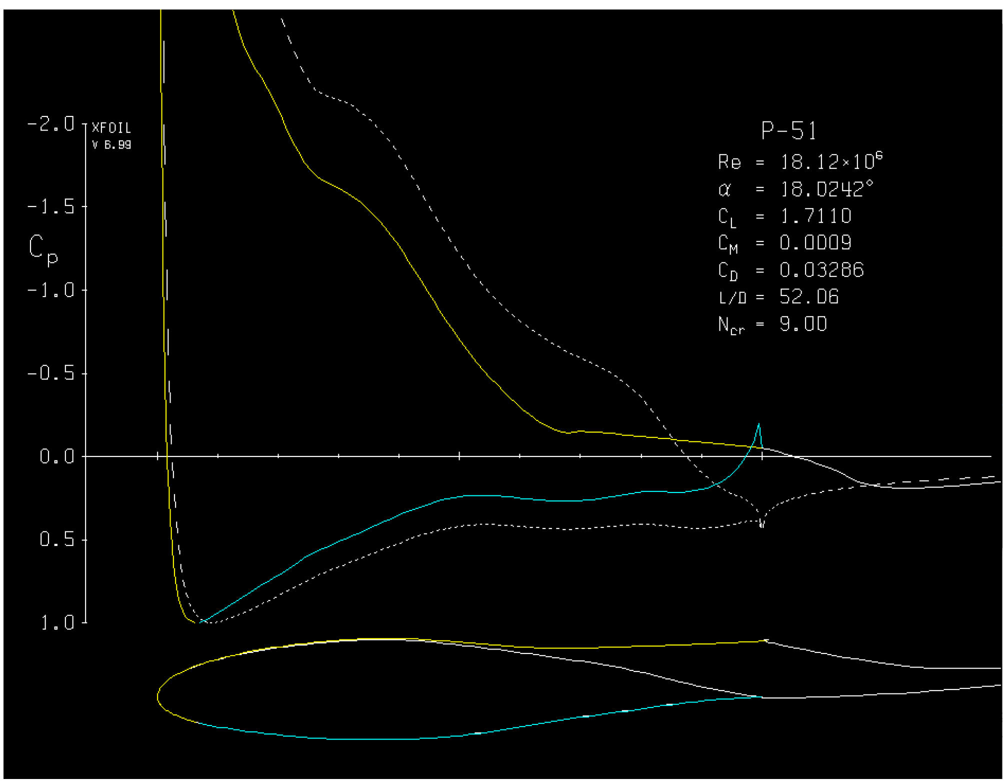

Figure 10: C_p distributions for stall flight with $C_L = 1.711$ on Xfoil

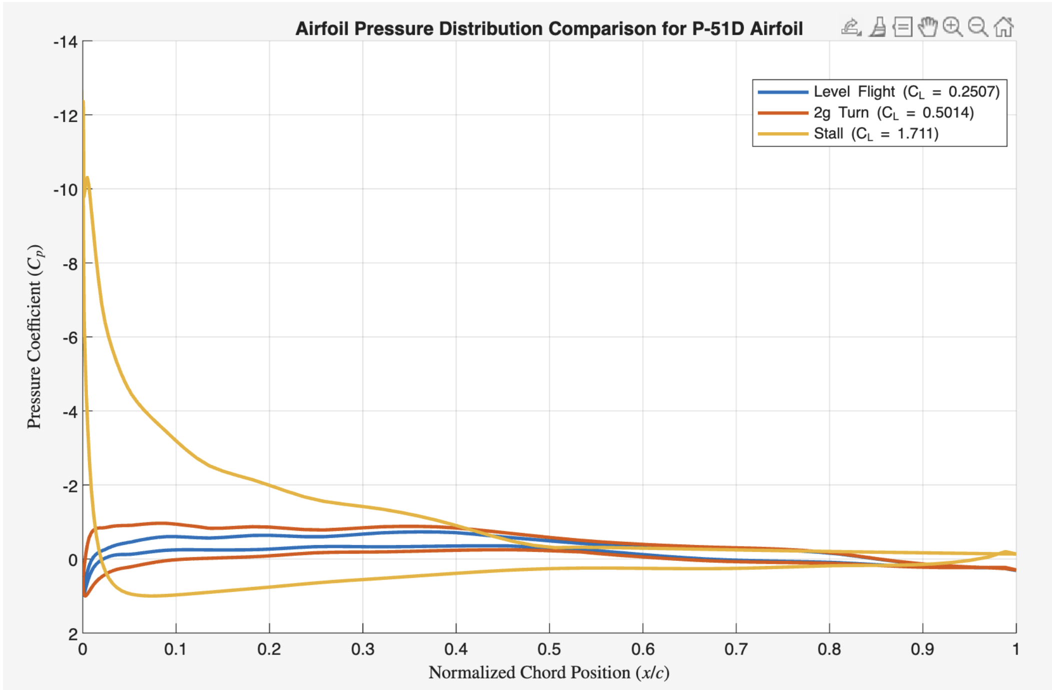

Figure 11: C_p distributions for level flight, $2g$ turn, and stall (P-51D airfoil).

Ans:

- Level-flight: $\alpha = 0.6332°$
- 2g-flight: $\alpha = 2.9350°$
- Stall-flight: $\alpha = 18.0242°$

The favorable pressure gradient strengthens as the suction peak becomes more negative with increasing lift coefficient. A deeper suction peak implies a higher local flow velocity over the forward section, necessary for greater lift generation.

As C_L approaches stall, the adverse pressure gradient steepens because the boundary layer must recover from a deeper suction peak toward the trailing-edge pressure over the same chord distance. Under stall conditions, the APG becomes too intense for the boundary layer to remain attached, resulting in flow separation.

In Xfoil, identifying the exact angle of attack at $C_{L,\max}$ was very difficult, since the low-fidelity nature of XFOIL kept producing inconsistent predictions of pressure gradients near stall.

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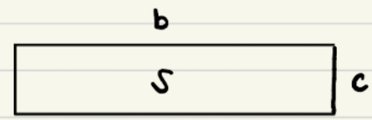
2. **3D Aircraft Performance**

Given a general aviation aircraft:

- Weight $W = 3000\,\text{lb}$
- Rectangular wing: $S = 170\,\text{ft}^2$, $b = 39\,\text{ft}$
- Airfoil: NACA 2412
- Empirical drag polar: $C_D = 0.025 + 0.054\,C_L^2$

2.



$$S = 170\ \text{ft}^2 \qquad AR = \frac{b^2}{S} = \frac{39^2}{170} = 8.94706$$

$$b = 39\ \text{ft}$$

$$c = \frac{S}{b} = 4.35897\ \text{ft}$$

$$V = 175\ \text{mph} \cdot \frac{1.46667\ \frac{\text{ft}}{\text{s}}}{1\ \text{mph}} = 256.6673\ \frac{\text{ft}}{\text{s}}$$

Sea level, Appendix E : $T = 518.69°R$, $P = 2.1162 \cdot 10^3\ \frac{\text{lb}}{\text{ft}^2}$

$$\rho = 2.3769 \cdot 10^{-3}\ \frac{\text{lb}}{\text{ft}^3}$$

$$Re = \frac{\rho V c}{\mu} \qquad \mu = 3.62 \cdot 10^{-7}\ \frac{\text{lb} \cdot \text{s}}{\text{ft}^2}$$

$$= \frac{2.3769 \cdot 10^{-3}\ \frac{\text{lb}}{\text{ft}^3} \cdot 256.6673\ \frac{\text{ft}}{\text{s}} \cdot 4.359\ \text{ft}}{3.62 \cdot 10^{-7}\ \frac{\text{lb} \cdot \text{s}}{\text{ft}^2}}$$

$$= 7.3461 \cdot 10^6$$

$$Ma = \frac{V}{a}$$ ∵ Assume it's ideal $P = \rho R T$

$$= \frac{V}{\sqrt{k \cdot \frac{P}{\rho}}} = \frac{256.667\ \frac{\text{ft}}{\text{s}}}{\sqrt{1.4 \cdot \frac{2.1162 \cdot 10^3\ \frac{\text{lb}}{\text{ft}^2}}{2.3769 \cdot 10^{-3}\ \frac{\text{lb}}{\text{ft}^3}}}} = 0.229897$$

Figure 12: Calculations for Re and Ma number.

Compute:

- Drag coefficient at zero lift $C_{D,0}$

- The drag coefficient at zero lift :

$$C_D = C_{D,0} + \frac{C_L^2}{\pi e AR}$$

$C_{D,i}$: induced drag coefficient

The parasite drag coefficient at zero lift

$$\begin{aligned} C_D &= 0.025 + 0.054 C_L^2 \\ &= 0.025 + 0.054 (0)^2 \\ &= 0.025 \end{aligned}$$

Figure 13: Calculations for C_D.

- Span efficiency factor e (derive from $C_{D,i} = C_L^2/(\pi ARe)$ and the given polar)

- The value of the span efficiency e :

$$C_{D,i} = \frac{C_L^2}{\pi e AR} = 0.054 C_L^2$$

$$\begin{aligned} e &= \frac{1}{\pi \cdot 0.054 \cdot 8.9420} \\ &= 0.65884 \end{aligned}$$

Figure 14: Calculations for e.

- C_D at $C_{L,\max}$ (obtain $C_{L,\max}$ from lift curve data)

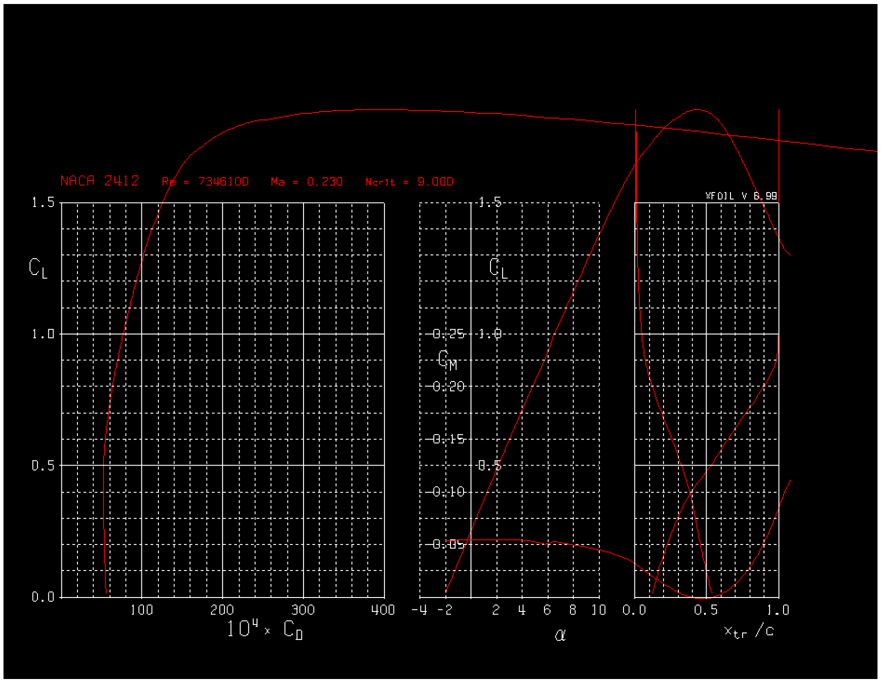


Figure 15: Lift curve of NACA 2412.

• C_D at $C_{L,max}$: From the lift curve on XFoil, $C_{L,max} = 1.852$

$$C_D = 0.025 + 0.054(1.852)^2$$
$$= 0.2102$$

Figure 16: Calculations for $C_{D,0}$.

- L/D at sea-level standard atmosphere and $V = 175\,\text{mph}$ level flight

• L/D

$$L = C_L \cdot q \cdot S \, , \quad D = C_D \cdot q \cdot S$$

$$\Rightarrow \frac{L}{D} = \frac{C_L}{C_D} \qquad C_L = \frac{3000 \text{ lb}}{\frac{1}{2} \cdot 2.3769 \cdot 10^{-3} \cdot 256.667^2 \cdot 170} = 0.22539$$

$$= \frac{0.22539}{0.02774} \qquad C_D = 0.025 + 0.054(0.2254)^2 = 0.02774$$

$$= \boxed{8.1244}$$

Figure 17: Calculations for L/D.

Ans:

- $C_D = 0.025$
- $e = 0.65884$
- C_D at $C_{L,max} = 0.2102$
- $L/D = 8.1244$

MATLAB Code

```
clear; clc; close all;

c = 8.48;

files  = {'n0012.dat.txt', 'naca23012.dat.txt', 'p51droot.dat'};
labels = {'NACA 0012', 'NACA 23012', 'P-51D Root'};
colors = {'r','b','g'};

figure('Color','w'); hold on; grid on;

for k = 1:numel(files)
    fname = files{k};

    xy = load(fname);
    x  = xy(:,1);
    y  = xy(:,2);

    le_idx = find(abs(x) < 1e-8 & abs(y) < 1e-8);

    if numel(le_idx) == 2
        xu = x(1:le_idx(1));
        yu = y(1:le_idx(1));

        xl = x(le_idx(2):end);
        yl = y(le_idx(2):end);

        xu = xu * c;  yu = yu * c;
        xl = xl * c;  yl = yl * c;

        plot(xu, yu, colors{k}, 'LineWidth', 1.5, 'DisplayName', labels{k});
        plot(xl, yl, colors{k}, 'LineWidth', 1.5, 'HandleVisibility','off');

    else
        x = x * c;
        y = y * c;
        plot(x, y, colors{k}, 'LineWidth', 1.5, 'DisplayName', labels{k});
    end
end

axis equal;
xlabel('x [ft]');
ylabel('y [ft]');
title('Airfoil Geometries in True Dimensions (c = 8.48 ft)');
legend('Location','best');
set(gca,'FontName','Times','FontSize',12);
```

Listing 1: 1.2 - Airfoil geometries

```
clear; clc; close all;

```

```
readPolar = @(fname) readmatrix(fname, "FileType","text", "NumHeaderLines",12);

P0012  = readPolar("polar_0012.txt");
P23012 = readPolar("polar_23012.txt");
PP51   = readPolar("test.txt");

a_0012  = P0012(:,1);   Cl_0012 = P0012(:,2);   Cd_0012 = P0012(:,3);
a_23012 = P23012(:,1);  Cl_23012 = P23012(:,2); Cd_23012 = P23012(:,3);
a_P51   = PP51(:,1);    Cl_P51 = PP51(:,2);     Cd_P51 = PP51(:,3);

LD_0012  = Cl_0012 ./ Cd_0012;
LD_23012 = Cl_23012 ./ Cd_23012;
LD_P51   = Cl_P51   ./ Cd_P51;

%CL vs alpha
figure('Color','w'); hold on;
plot(a_0012,  Cl_0012,  'b-',  'LineWidth',1.5);
plot(a_23012, Cl_23012, 'r-',  'LineWidth',1.5);
plot(a_P51,   Cl_P51,   'k-',  'LineWidth',1.5);

xlabel('\alpha [deg]');
ylabel('C_L');
title('Viscous Lift Curves');
legend('NACA 0012','NACA 23012','P-51D root','Location','best');
grid on;

%Cl vs Cd zoom
figure('Color','w'); hold on;
plot(Cl_0012,  Cd_0012,  'b-', 'LineWidth',1.5);
plot(Cl_23012, Cd_23012, 'r-', 'LineWidth',1.5);
plot(Cl_P51,   Cd_P51,   'k-', 'LineWidth',1.5);

xlabel('C_L');
ylabel('C_D');
title('Viscous Drag Polars (Zoomed Drag Bucket View)');
legend('NACA 0012','NACA 23012','P-51D root','Location','northwest');
grid on;
axis([-2 2 0 0.025]);

%Cl vs Cd
figure('Color','w'); hold on;
plot(Cl_0012,  Cd_0012,  'b-', 'LineWidth',1.5);
plot(Cl_23012, Cd_23012, 'r-', 'LineWidth',1.5);
plot(Cl_P51,   Cd_P51,   'k-', 'LineWidth',1.5);

xlabel('C_L');
ylabel('C_D');
title('Viscous Drag Polars (Full Range)');
legend('NACA 0012','NACA 23012','P-51D root','Location','northwest');
grid on;

%L/D vs alpha
```

```matlab
figure('Color','w'); hold on;
plot(a_0012,  LD_0012,  'b-', 'LineWidth',1.5);
plot(a_23012, LD_23012, 'r-', 'LineWidth',1.5);
plot(a_P51,   LD_P51,   'k-', 'LineWidth',1.5);

xlabel('\alpha␣[deg]');
ylabel('L/D');
title('Lift-to-Drag␣Ratio␣vs␣Angle␣of␣Attack');
legend('NACA␣0012','NACA␣23012','P-51D␣root','Location','best');
grid on;
```

Listing 2: 1.2 - Viscous lift curves, Viscous drag polars in two plots, and Lift-to-Drag ratio vs AoA

```matlab

fileNames = {
    'Cp_level_flight.dat', ...
    'Cp_2g_flight.dat', ...
    'Cp_stall_flight.dat'
};

plotLabels = {
    'Level␣Flight␣(C_L␣=␣0.2507)', ...
    '2g␣Turn␣(C_L␣=␣0.5014)', ...
    'Stall␣(C_L␣=␣1.711)'
};

dataSets = cell(1, 3);

for i = 1:3
    filename = fileNames{i};

    fid = fopen(filename, 'r');

    fgetl(fid); % Skip line 1
    fgetl(fid); % Skip line 2
    fgetl(fid); % Skip line 3

    data_cell = textscan(fid, '%f␣%f␣%f', 'CollectOutput', true);

    fclose(fid);

    dataSets{i} = data_cell{1};
end


figure;
hold on;
grid on;

for i = 1:3
    data_matrix = dataSets{i};

    x_data = data_matrix(:,1);
    Cp_data = data_matrix(:,3);
```

```
    plot(x_data, Cp_data, 'LineWidth', 2, 'DisplayName', plotLabels{i});
end

set(gca, 'YDir', 'reverse');
xlabel('Normalized Chord Position ($x/c$)', 'Interpreter', 'latex');
ylabel('Pressure Coefficient ($C_p$)', 'Interpreter', 'latex');
title('Airfoil Pressure Distribution Comparison for P-51D Airfoil');
legend('show', 'Location', 'SouthWest');
xlim([0 1]);
hold off;
```

Listing 3: 1,3 - $C_p plotsforeachairfoil$

```
P0012 = readmatrix("2412_polar.txt","FileType","text","NumHeaderLines",12);

a  = P0012(:,1);
Cl = P0012(:,2);
Cd = P0012(:,3);

[Cl_stall, i_stall] = max(Cl);

alpha_stall = a(i_stall);

fprintf('Stall Cl  = %.3f\n', Cl_stall);
```

Listing 4: 2 - $C_{L,max}$ calculation for P-51D and NACA 2412