EAE 129 WQ 2025 Midterm Project Report

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1 Abstract

In our wind tunnel experiment, we tested and measured the longitudinal static stability characteristics of the Aggie UAV. The lift and moment coefficients were recorded across a range of angles of attack from -5 to 30 degrees, at three different elevator deflections: -5, 0, and 5 degrees. From the six data tables obtained, we used linear averaging to derive the aerodynamic and stability derivatives for each graph. With these derived values and the given geometric characteristics of the Aggie, we estimated the Stability Margin, Neutral Point, and derived an expression for the trimmed elevator angle.

Contents

2 Introduction

2.1 Purpose

A fundamental principle in aerodynamics and flight dynamics is that aircraft undergo extensive testing, using prototype models, to evaluate their performance before first flight. These prototypes, commonly known as mock-ups, are designed with similar aerodynamic characteristics to the final aircraft. In our wind tunnel experiment, our goal is to visualize the aerodynamic characteristics of the Aggie UAV and to provide experimental results that other engineers can reproduce on thier own with data and derivations provided on this report.

2.2 Governing Equations

- This equation models how the lift coefficient changes due to the aircraft's angle of attack and control surface deflection angle.

  $$C_L = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{\delta_e}}\delta_e \tag{1}$$

  where C_L is the lift coefficient for a finite wing, C_{L_0} (Zero-lift coefficient) is the lift coefficient when the angle of attack and the elevator angle are both zero, C_{L_α} (Lift curve slope) is the rate of change of the lift coefficient with respect to the angle of attack, and $C_{L_{\delta_e}}$ is the rate of change of the lift coefficient with respect to the elevator deflection angle.

- This equation is important for analyzing aircraft stability and control, particularly in determining how an aircraft responds to changes in angle of attack.

  $$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e \tag{2}$$

  where C_m is the pitching moment coefficient for a finite wing, C_{m_0} (Zero-angle pitching moment coefficient) is the pitching moment coefficient when the angle of attack and the elevator angle are both zero, C_{m_α} (Pitching moment slope due to angle of attack) is the rate of change of the pitching moment coefficient with respect to the angle of attack, and $C_{m_{\delta_e}}$ is the rate of change of the pitching moment coefficient with respect to elevator deflection.

- C_{m_α} determines the aircraft's longitudinal static stability. For stability, this value should be negative. SM is a dimensionless measure of how far the Center of Gravity (CG) is located ahead of the Neutral Point (NP)

  $$C_{m_\alpha} = -SM * C_{L_\alpha} \tag{3}$$

  A positive static margin means the CG is ahead of the NP, ensuring longitudinal static stability.

- This equation is set equal to zero, indicating that the sum of the pitching moments from the wing and tail must balance for the aircraft to be in equilibrium.

$$C_{m_\alpha} = C_{L_{\alpha w}} * (\overline{x}_{cg} - \overline{AC}_w) - C_{L_{\alpha_h}} * \eta_h * \frac{S_h}{S} * (\overline{x}_{AC_h} - \overline{x}_{cg}) * (1 - \frac{d\epsilon}{d\alpha}) \tag{4}$$

where $\overline{x}_{cg}$ is a location of the center of gravity (CG) of the aircraft, $\overline{AC}_w$ is the aerodynamic center of a wing, at which the pitching moment is constant with respect to changes in angle of attack, η_h is a factor that scales the horizontal tail's contribution to the aircraft's overall lift, S and S_h are surface area of a wing and horizontal tail, respectively, and $\frac{d\epsilon}{d\alpha}$ shows how the downwash changes with the angle of attack. This equation holds true for total pitching moment about center of gravity and crucial for the aircraft's stability control.

3 Results

3.1 Plots

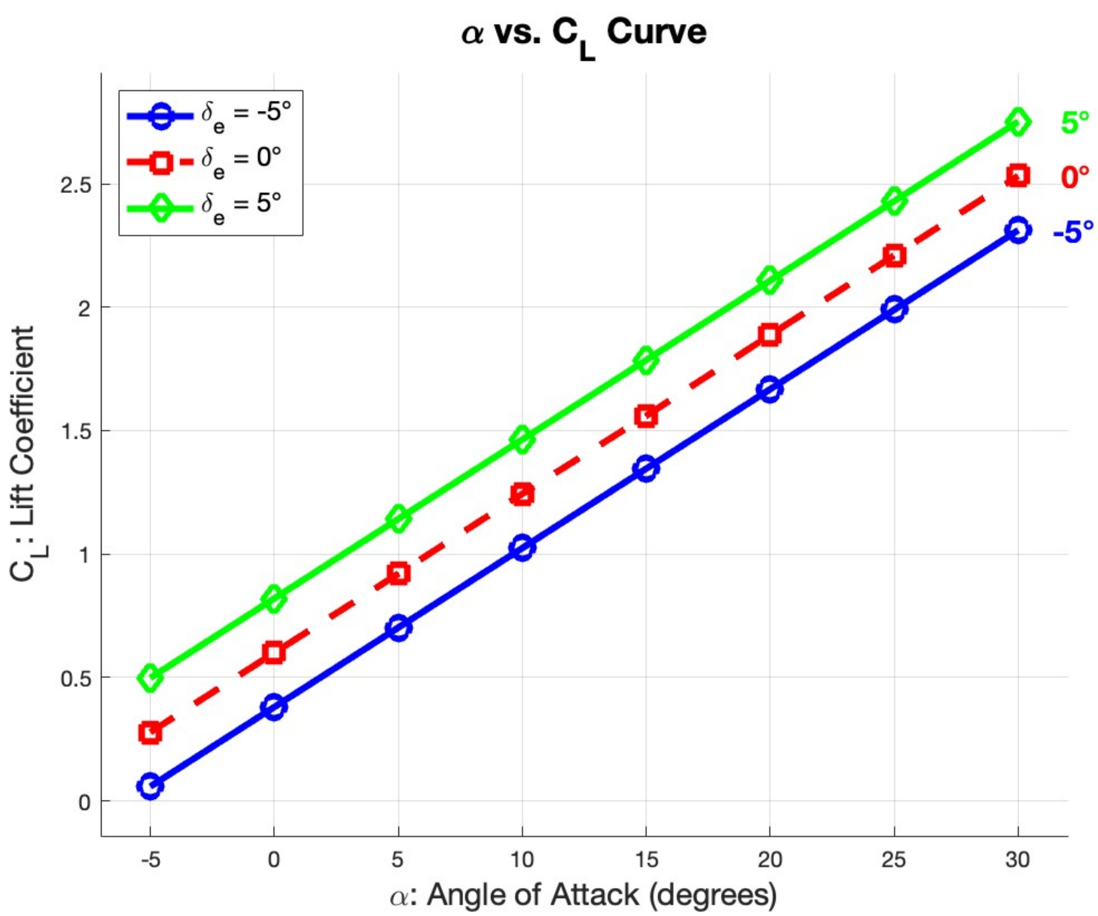

Figure 1: Plot of C_L versus α for all three elevator angles

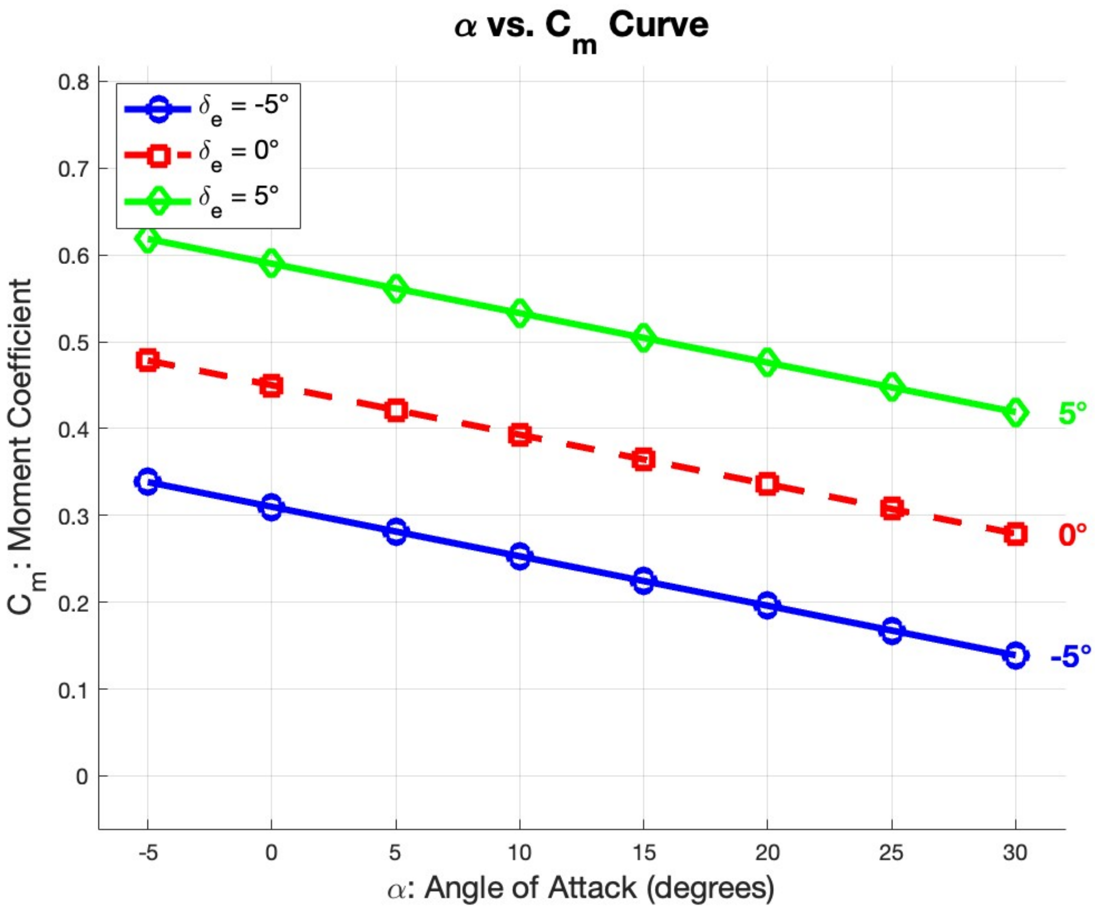

Figure 2: Plot of C_m versus α for all three elevator angles

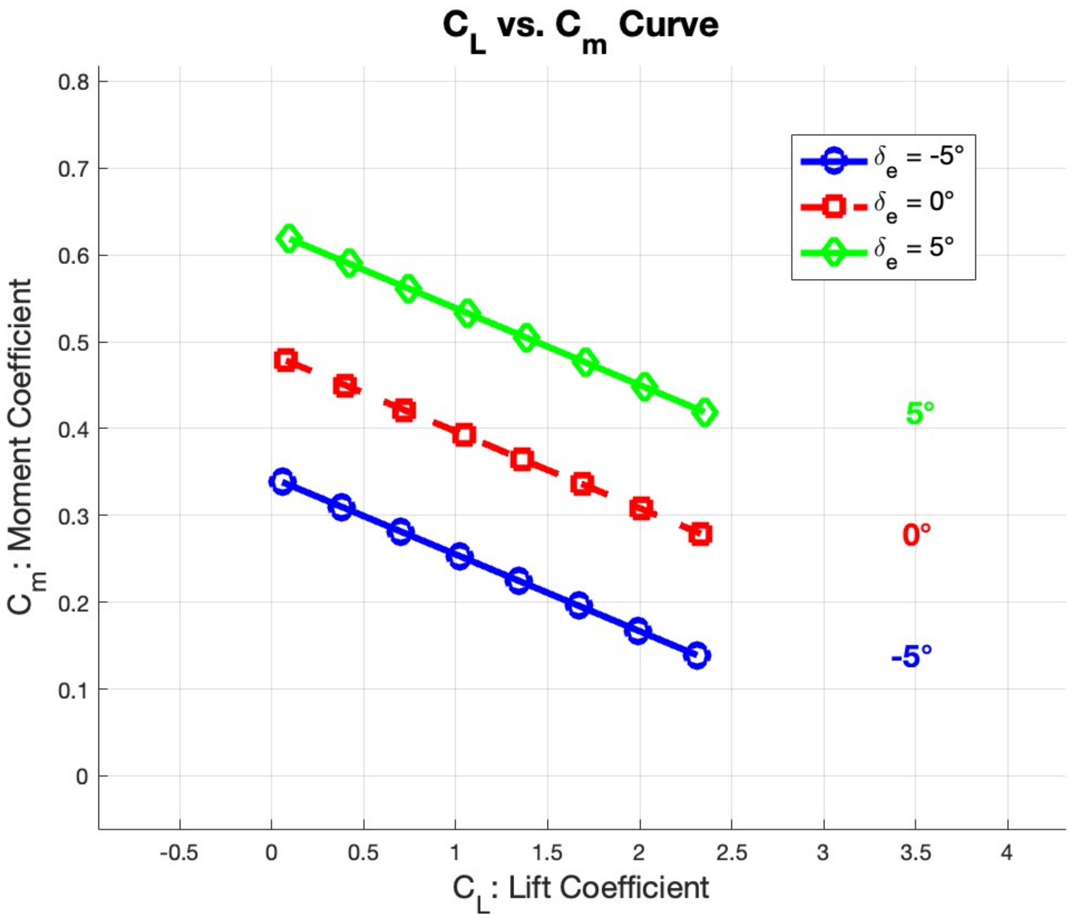

Figure 3: Plot of C_m versus C_L for all three elevator angles

4 Discussion

4.1 Aerodynamic and Stability Derivatives Estimates

- From the Figure 1 above, C_L versus α graphs are linear. Therefore, we can take a derivative of each linear graph by taking an average of each point.

δ_e	C_{L_α}
-5	0.0644
0	0.0644
5	0.0644

Table 1: C_{L_α} obtained from graphs

$$C_L = C_{L_0} + C_{L_\alpha}\alpha + C_{L_{\delta_e}}\delta_e(1)$$

In this first-order differential equation, only two variables are unknown, which are C_{L_0} and $C_{L_{\delta_e}}$. Since we have data of C_L when $\delta_e = 0$, we can calculate the value of C_{L_0} first. For convenience, from Table 6, if I choose the C_L for $\alpha = 0$, I can directly get $C_{L_0} = 0.400$. Now, I plug the C_{L_0} into the equation with other known values chosen from Table 6 or 7 data sets. I chose $C_L = 1.0240$ and α=10 from Table 5, and the equation gave me $C_{L_{\delta_e}} = 0.004$.

As a result, we estimate that C_{L_0} = 0.400, C_{L_α} = 0.0644, $C_{L_{\delta_e}}$ = 0.004.

- Same method applies to C_{m_α}. As Figure 3 shows above, each curve is linear. Therefore, we can estimate the slope of each trend by taking an average.

δ_e	C_{m_α}
-5	-0.0057
0	-0.0057
5	-0.0057

Table 2: C_{m_α} obtained from graphs

$$C_m = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e(2)$$

Just as above, in this first-order differential equation, only two variables are unknown, which are C_{m_0} and $C_{m_{\delta_e}}$. Since we have data of C_m when $\delta_e = 0$, we can calculate the value of C_{m_0} first. For convenience, from Table 9, if I choose the C_m for $\alpha = 0$, I can directly get $C_{m_0} = 0.1900$. Now, I plug the C_{m_0} into the equation with other known values chosen from Table 8 or 10 data sets. I chose $C_m = 0.2530$, α=10 from Table 8, and the equation gave me $C_{m_{\delta_e}}$ = -0.024.

As a result, we estimate that C_{m_0} = 0.1900, C_{m_α} = -0.0057, $C_{m_{\delta_e}}$ = -0.024.

4.2 Static Stability Analysis

- To obtain the stability margin (SM), you need both C_m and C_L.

$$C_{m_\alpha} = -SM * C_{L_\alpha}(3)$$

δ_e	$\frac{C_m}{C_L}$
-5	-0.0885
0	-0.0885
5	-0.0885

Table 3: C_m/C_L obtained from graphs

Therefore, we estimate that SM = 0.0885 and is stable

- In static longitudinal control, when the aircraft is trimmed, the pitching moment coefficient is 0.

$$C_m = C_{m_0} + C_{m_\alpha} * \alpha_{trim} + C_{m_{\delta_e}} * \delta_e(2)$$

Plugging in the values obtained above, you can obtain a direct relationship between δ_e and α_{trim}. As a result,

$$\delta_{e_{trim}} = 79.1667 + 2.375 * \alpha_{trim} \tag{5}$$

- By definition, a Neutral Point (NP) is a point where an aircraft is neutrally stable if the center of gravity is placed. Therefore, when an aircraft is neutrally stable, an Aircraft Dynamic Center (AC) and NP are located at the same point for the same aircraft.

$$C_{m_\alpha} = C_{L_{\alpha_w}} * (\overline{x}_{cg} - \overline{AC}_w) - C_{L_{\alpha_h}} * \eta_h * \frac{S_h}{S} * (\overline{x}_{AC_h} - \overline{x}_{cg}) * (1 - \frac{d\epsilon}{d\alpha}) = 0(4)$$

Solve for $\overline{x}_{cg} = \overline{x}_{AC}$

Let us call

$$\frac{C_{L_{\alpha_h}}}{C_{L_{\alpha_w}}}\eta_h\frac{S_h}{S}\left(1 - \frac{d\epsilon}{d\alpha}\right) = B \tag{6}$$

If we combine Equation (4) and (6), we can get

$$\overline{x}_{AC} = \frac{\overline{x}_{AC_w} + B\overline{x}_{AC_h}}{1 + B} \tag{7}$$

All the information regarding the geometry of the aircraft is given:

$\overline{c}_w$	6 in
$\overline{c}_h$	6 in
S_w	150 in^2
S_h	30 in^2
$\overline{x}_{cg}$	1.0
$\overline{x}_{AC_h}$	4.0
$\overline{x}_{AC_w}$	0.25
η_h	1.0
$\frac{d\epsilon}{d\alpha}$	0
AR_w	1.676
AE_h	3.82
e	1.0

Table 4: Geometry of the aircraft

Since the wing and horizontal tail have the same cord length, they have the same lift coefficient as well. Therefore, we can use the C_{L_α} obtained above. Calculating B from the values given in Table 4, we get $\mathbf{B = 0.2}$. Plugging in B and other variables to Equation (7), **we estimate that** $\overline{x}_{NP} = \mathbf{0.875}$

5 Conclusion

The linear relationships observed in the C_L versus α and C_m versus α graphs allowed us to calculate the aerodynamic derivatives, C_{L_α} and C_{m_α}, which were important in evaluating the stability of the aircraft. Based on our estimates and plots, the stability margin was found to be positive, confirming that the Aggie UAV maintains fair static longitudinal stability within the tested range of elevator angles. Additionally, we calculated the neutral point of the aircraft, which serves as another important parameter for understanding the aircraft's stability and its behavior in different flight conditions.

If we were to improve the performance of the Aggie, we could consider moving the center of gravity forward and increase SM. A bigger and positive SM ensures that aircraft is less sensitive to perturbations and more stable. One of the assumption and design limit we had was that the wing and horizontal tail are identical. If we changed the tail volume ratio, the horizontal tail would play more significant role in control and effectively increase the SM. We also want to consider enhance C_{L_α} value as higher C_{L_α} will provide greater lift at lower angles of attack. For the future experiments, we could conduct more testings to refine the aerodynamic characterstic data and the aircraft's performance.

A Appendix

A.1 Wind Tunnel Data

α (deg)	C_L
-5	0.0580
0	0.3800
5	0.7020
10	1.0240
15	1.3460
20	1.6680
25	1.9900
30	2.3120

Table 5: Change of lift coefficient (C_L) with respect to different angles of attack (α) when elevator angle δ_e = -5 deg.

α (deg)	C_L
-5	0.0780
0	0.4000
5	0.7220
10	1.0440
15	1.3660
20	1.6880
25	2.0100
30	2.3320

Table 6: Change of lift coefficient (C_L) with respect to different angles of attack (α) when elevator angle $\delta_e = 0$ deg.

α (deg)	C_L
-5	0.0980
0	0.4200
5	0.7420
10	1.0640
15	1.3860
20	1.7080
25	2.0300
30	2.3520

Table 7: Change of lift coefficient (C_L) with respect to different angles of attack (α) when elevator angle $\delta_e = 5$ deg.

α (deg)	C_m
-5	0.3385
0	0.3100
5	0.2815
10	0.2530
15	0.2245
20	0.1960
25	0.1675
30	0.1390

Table 8: Change of lift coefficient (C_m) with respect to different angles of attack (α) when elevator angle δ_e = -5 deg.

α (deg)	C_m
-5	0.2185
0	0.1900
5	0.1615
10	0.1330
15	0.1045
20	0.0760
25	0.0475
30	0.0190

Table 9: Change of lift coefficient (C_m) with respect to different angles of attack (α) when elevator angle $\delta_e = 0$ deg.

α (deg)	C_m
-5	0.2785
0	0.2500
5	0.2215
10	0.1930
15	0.1645
20	0.1360
25	0.1075
30	0.0790

Table 10: Change of lift coefficient (C_m) with respect to different angles of attack (α) when elevator angle $\delta_e = 5$ deg.

A.2 MATLAB Code

Below is the MATLAB code used for the plots:

```
% Angle of Attack Inputs
data1 = [-5,0,5,10,15,20,25,30];

% C_L Data
data2 = [0.0580,0.3800,0.7020,1.0240,1.3460,1.6680,1.9900,2.3120];
data3 = [0.0780,0.4000,0.7200,1.0440,1.3600,1.6880,2.0100,2.3320];
data4 = [0.0980,0.4200,0.7420,1.0640,1.3860,1.7080,2.0300,2.3520];

% C_m Data
data5 = [0.3385,0.3100,0.2815,0.2530,0.2245,0.1960,0.1675,0.1390];
data6 = [0.2785,0.2500,0.2215,0.1930,0.1645,0.1360,0.1075,0.0790];
data7 = [0.2185,0.1900,0.1615,0.1330,0.1045,0.0760,0.0475,0.0190];

% Offset for spacing (the graphs were initially shown very close to each other)
offset = 0.2; %arbituary but it came out great
data2_shifted = data2;
data3_shifted = data3 + offset;
data4_shifted = data4 + 2*offset;

data5_shifted = data5;
data6_shifted = data6 + offset;
data7_shifted = data7 + 2*offset;

% First Figure: C_L vs. Alpha
figure;
hold on;
plot(data1, data2_shifted, 'b-o', 'LineWidth', 3, 'MarkerSize', 10, 'DisplayName', '\delta_e = -5  ');
plot(data1, data3_shifted, 'r--s', 'LineWidth', 3, 'MarkerSize', 10, 'DisplayName', '\delta_e = 0  ');
plot(data1, data4_shifted, 'g-d', 'LineWidth', 3, 'MarkerSize', 10, 'DisplayName', '\delta_e = 5  ');

% Labels for C_L
text(data1(end) + 1, data2_shifted(end), ' -5  ', 'Color', 'b', 'FontSize', 14, 'FontWeight', 'bold');
text(data1(end) + 1, data3_shifted(end), '  0  ', 'Color', 'r', 'FontSize', 14, 'FontWeight', 'bold');
text(data1(end) + 1, data4_shifted(end), '  5  ', 'Color', 'g', 'FontSize', 14, 'FontWeight', 'bold');

xlabel('\alpha: Angle of Attack (degrees)', 'FontSize', 14);
ylabel('C_L: Lift Coefficient', 'FontSize', 14);
title('\alpha vs. C_L Curve', 'FontSize', 16);
legend('Location', 'northwest', 'FontSize', 12);
grid on;

%Leave some margin for readiness
ylim([min(data2)-0.2, max(data4_shifted)+0.2]);
xlim([min(data1)-2, max(data1)+2]);

hold off;
```

```
% Second Figure: C_m vs. Alpha
figure;
hold on;
plot(data1, data5_shifted, 'b-o', 'LineWidth', 3, 'MarkerSize', 10, 'DisplayName', '\delta_e␣=␣-5 ');
plot(data1, data6_shifted, 'r--s', 'LineWidth', 3, 'MarkerSize', 10, 'DisplayName', '\delta_e␣=␣0 ');
plot(data1, data7_shifted, 'g-d', 'LineWidth', 3, 'MarkerSize', 10, 'DisplayName', '\delta_e␣=␣5 ');

% Labels for C_m
text(data1(end) + 1, data5_shifted(end), '␣-5 ', 'Color', 'b', 'FontSize', 14, 'FontWeight', 'bold');
text(data1(end) + 1, data6_shifted(end), '␣␣0 ', 'Color', 'r', 'FontSize', 14, 'FontWeight', 'bold');
text(data1(end) + 1, data7_shifted(end), '␣␣5 ', 'Color', 'g', 'FontSize', 14, 'FontWeight', 'bold');

xlabel('\alpha:␣Angle␣of␣Attack␣(degrees)', 'FontSize', 14);
ylabel('C_m:␣Moment␣Coefficient', 'FontSize', 14);
title('\alpha␣vs.␣C_m␣Curve', 'FontSize', 16);
legend('Location', 'northwest', 'FontSize', 12);
grid on;

ylim([min(data5)-0.2, max(data7_shifted)+0.2]);
xlim([min(data1)-2, max(data1)+2]);

hold off;


% Third Figure: C_m vs. C_L
figure;
hold on;
plot(data2, data5_shifted, 'b-o', 'LineWidth', 3, 'MarkerSize', 10, 'DisplayName', '\delta_e␣=␣-5 ');
plot(data3, data6_shifted, 'r--s', 'LineWidth', 3, 'MarkerSize', 10, 'DisplayName', '\delta_e␣=␣0 ');
plot(data4, data7_shifted, 'g-d', 'LineWidth', 3, 'MarkerSize', 10, 'DisplayName', '\delta_e␣=␣5 ');

% Labels
text(data2(end) + 0.3, data5_shifted(end), '␣-5 ', 'Color', 'b', 'FontSize', 14, 'FontWeight', 'bold');
text(data3(end) + 0.3, data6_shifted(end), '␣␣0 ', 'Color', 'r', 'FontSize', 14, 'FontWeight', 'bold');
text(data4(end) + 0.3, data7_shifted(end), '␣␣5 ', 'Color', 'g', 'FontSize', 14, 'FontWeight', 'bold');

xlabel('C_L:␣Lift␣Coefficient', 'FontSize', 14);
ylabel('C_m:␣Moment␣Coefficient', 'FontSize', 14);
title('C_L␣vs.␣C_m␣Curve', 'FontSize', 16);
legend('Location', 'northwest', 'FontSize', 12);
```

```
grid on;

ylim([min(data5)-0.2, max(data7_shifted)+0.2]);
xlim([min(data2)-1, max(data2)+2]);

hold off;

% Slope for delta_e = -5
slope_C_L_1 = diff(data2) ./ diff(data1);
% Slope for delta_e = 0
slope_C_L_2 = diff(data3) ./ diff(data1);
% Slope for delta_e = 5
slope_C_L_3 = diff(data4) ./ diff(data1);

fprintf('C_L_alpha_1␣=␣%d\n', slope_C_L_1);
fprintf('C_L_alpha_2␣=␣%d\n', slope_C_L_2);
fprintf('C_L_alpha_3␣=␣%d\n', slope_C_L_3);

% Slope for delta_e = -5
slope_C_m_1 = diff(data5) ./ diff(data1);
% Slope for delta_e = 0
slope_C_m_2 = diff(data6) ./ diff(data1);
% Slope for delta_e = 5
slope_C_m_3 = diff(data7) ./ diff(data1);

fprintf('C_m_alpha_1␣=␣%d\n', slope_C_m_1);
fprintf('C_m_alpha_2␣=␣%d\n', slope_C_m_2);
fprintf('C_m_alpha_3␣=␣%d\n', slope_C_m_3);

% Slope for delta_e = -5
slope_C_m_C_L_1 = diff(data5) ./ diff(data2);
% Slope for delta_e = 0
slope_C_m_C_L_2 = diff(data6) ./ diff(data3);
% Slope for delta_e = 5
slope_C_m_C_L_3 = diff(data7) ./ diff(data4);

fprintf('C_m_C_L_1␣=␣%d\n', slope_C_m_C_L_1);
fprintf('C_m_C_L_2␣=␣%d\n', slope_C_m_C_L_2);
fprintf('C_m_C_L_3␣=␣%d\n', slope_C_m_C_L_3);
```

Listing 1: External MATLAB Script