EAE 135 Project 3

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Contents

1 Deliverable 1: Mathematical Derivations

1. Calculate the coordinate of the modulus-weighted centroid. Ignore the skin in calculations as stated in section 8.10.2 of Bauchau and Craig (B and C). Report its location relative to point O in Figure 1. Tabulate the values in your report.

   From Figure 2 in Appendix, we can obtain Young's modulus, E, for the material aluminum T861 as $10.5 * 10^3$ ksi.

   Equations needed to calculate the modulus-weighted centroid for x_2 and x_3 are:

$$x_2^c = \frac{\Sigma E^{(i)} A^{(i)} x_2^{(i)}}{\Sigma E^{(i)} A^{(i)}} \tag{1}$$

$$x_3^c = \frac{\Sigma E^{(i)} A^{(i)} x_3^{(i)}}{\Sigma E^{(i)} A^{(i)}} \tag{2}$$

$$\begin{aligned} E &= 10.5 \cdot 10^3 \text{ ksi} \\ &= 10.5 \cdot 10^3 \text{ ksi} \cdot \frac{0.00689476 \text{ GPa}}{\text{ksi}} \\ &\approx 72.39498 \text{ GPa} \end{aligned}$$

$$\begin{aligned} A_s &= 150 \text{ mm}^2 \\ &\approx 0.00015 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} x_2^c &= \frac{\Sigma_i E^{(i)} \cdot A^{(i)} \cdot x_2^{(i)}}{\Sigma_i E^{(i)} \cdot A^{(i)}} \\ &= \frac{\cancel{72.39498} \cdot \cancel{0.00015} \cdot \left(7R + R + 0 - R - \frac{2}{3}R + R\right)}{6 \cdot \cancel{72.39498} \cdot \cancel{0.00015}} \\ &= 1.2222 \cdot R \\ &= \frac{11}{9} \cdot R \end{aligned}$$

$$\begin{aligned} R : x_3^{(2)} &= 7R : 6R \\ 7R x_3^{(2)} &= 6R^2 \\ x_3^{(2)} &= \frac{6}{7}R \end{aligned}$$

$$\begin{aligned} x_3^c &= \frac{\Sigma_i E^{(i)} A^{(i)} \cdot x_3^{(i)}}{\Sigma_i E^{(i)} \cdot A^{(i)}} \\ &= \frac{0 + \frac{6}{7}R + R - \frac{1}{3}R - \frac{6}{23}R}{6} \\ &= \frac{305}{1449} R \end{aligned}$$

$$\begin{aligned} \frac{R}{3} : x_3^{(6)} &= \left(8R - \frac{1}{3}R\right) : 6R \\ \frac{23}{3}R \cdot x_3^{(6)} &= 2R^2 \\ x_3^{(6)} &= \frac{2R^2}{\frac{23}{3}R} \\ &\approx \frac{6}{23}R \end{aligned}$$

Plugging in the value 1.95 m (8) into R, we can obtain:

x_c^2	x_c^3
2.3833 m	0.41046 m

Table 1: Modulus Weighted Centroid

Therefore, the new reference system in terms of the modulus-weighted centroid is:

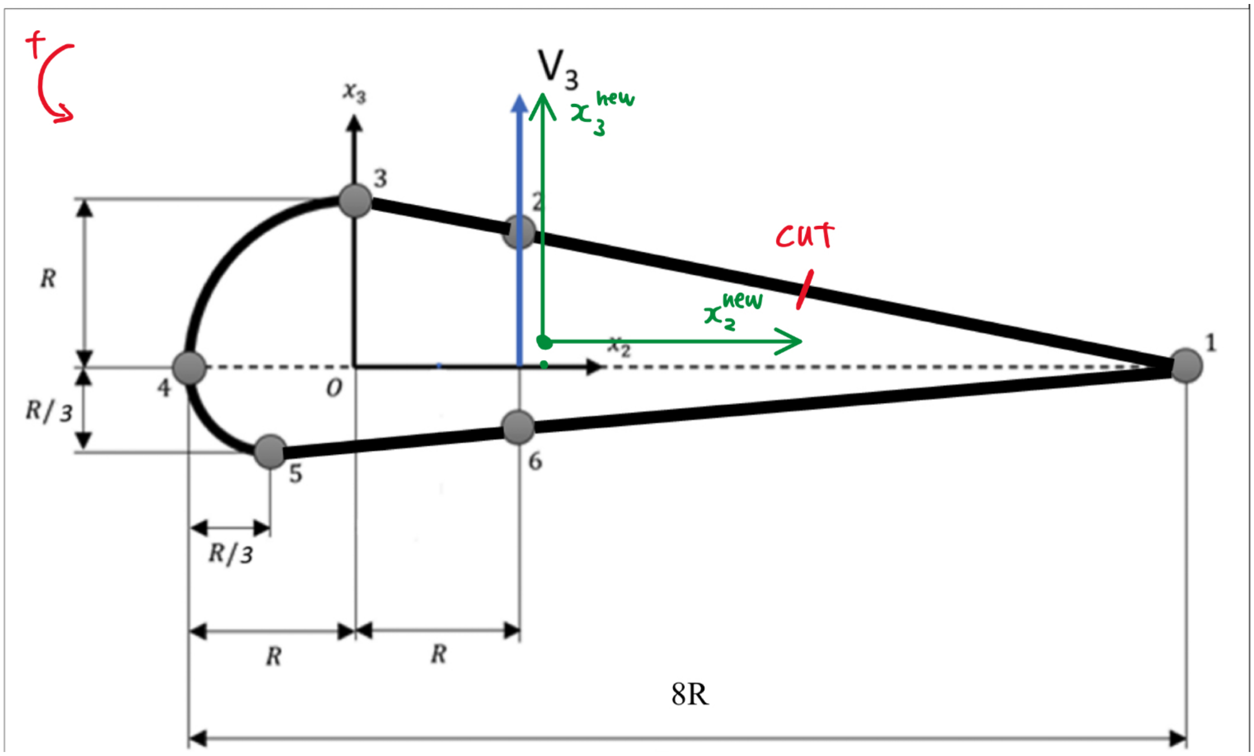


Figure 1: Wing Cross-Section and Loading Model with a new reference system (NOT TO SCALE)

2. Calculate H_{22}^c, H_{33}^c , and H_{23}^c in terms of the modulus-weighted centroid. Ignore the skin contributions in the calculations.

   Equations for bending stiffness, H_{22}^c and H_{33}^c, and a cross-bending stiffness H_{23}^c are:

   You have to note that x_3 and x_2 in equations are with respect to the modulus-weighted centroid (1).

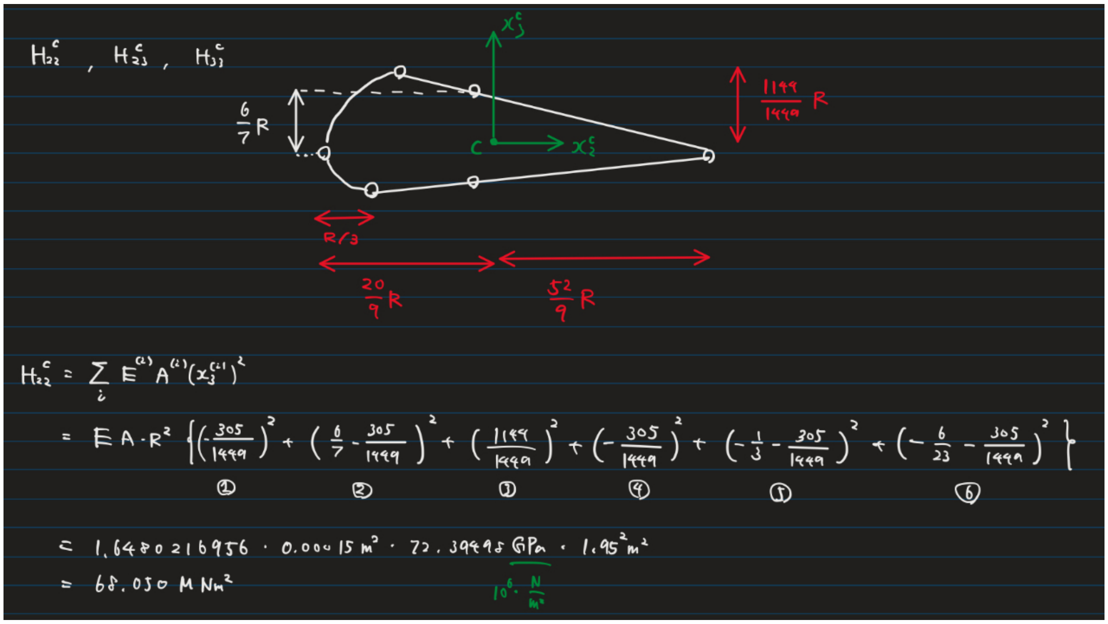
$$H_{22}^c = \Sigma E^{(i)} A^{(i)} (x_3^{(i)})^2 \tag{3}$$

$$H_{33}^c = \Sigma E^{(i)} A^{(i)} (x_2^{(i)})^2 \tag{4}$$

$$H_{23}^c = \Sigma E^{(i)} A^{(i)} (x_2^{(i)})(x_3^{(i)}) \tag{5}$$

Calculations are below:

(a) H_{22}^c



(b) H_{33}^c

$$H_{33}^c = \sum_i E^{(i)} A^{(i)} (x_2^{(i)})^2$$

$$= EAR^2 \left\{ \left(\frac{52}{9}\right)^2 + \left(-\frac{11}{9} + 1\right)^2 + \left(-\frac{11}{9}\right)^2 + \left(-\frac{20}{9}\right)^2 + \left(-\frac{2}{3} - \frac{11}{9}\right)^2 + \left(-\frac{11}{9} + 1\right)^2 \right\}$$

① ② ③ ④ ⑤ ⑥

$$= 43.4815 \cdot 0.00015\ m^2 \cdot 72.39498\ GPa \cdot 1.95^2\ m^2$$

$$= 1.79545\ GNm^2$$

(c) H_{23}^c

$$H_{23}^{c} = \sum_{i} E^{(i)} A^{(i)} x_2^{(i)} \cdot x_3^{(i)}$$

$$= E \cdot A \cdot R^2 \Big\{ \underset{①}{\left(-\frac{305}{1449} \cdot \frac{52}{9}\right)} + \underset{②}{\left(\frac{6}{7} - \frac{305}{1449}\right)\left(-\frac{11}{9}+1\right)} + \underset{③}{\left(\frac{1144}{1449}\right)\left(-\frac{11}{9}\right)} + \underset{④}{\left(-\frac{305}{1449}\right)\left(-\frac{20}{9}\right)}$$

$$+ \underset{⑤}{\left(-\frac{1}{3} - \frac{305}{1449}\right)\left(-\frac{2}{3} - \frac{11}{9}\right)} + \underset{⑥}{\left(-\frac{6}{23} - \frac{305}{1449}\right)\left(-\frac{11}{9}+1\right)} \Big\}$$

$$= EAR^2 \left(-1.216164 - 0.143701 - 0.964957 + 0.4677555 + 1.027222 + 0.1047466\right)$$

$$= 72.39498 \cdot GPa \cdot 0.00015 m^2 \cdot 1.95^2 m^2 \cdot -0.7250979$$

$$= -0.029940 \cdot G\, Nm^2$$

$$= -2.994 \cdot 10^7\ Nm^2$$

H_{22}^c	H_{33}^c	H_{23}^c
6.8050 * 10^7 Nm^2	1.79545 * 10^9 Nm^2	-2.994 * 10^7 Nm^2

Table 2: Bending Stiffness in terms of the modulus-weighted centroid

3. **Make the cut to open the cross-section between stringers 1 and 2.** Calculate the open shear flow f_{oij} in each skin segment.

   The jump of shear flow due to the presence of stringer (i) is given by:

$$\triangle f^{(i)} = -E_{(i)} A_{(i)} \left[\frac{H_{22}^c V_2 - H_{23}^c V_3}{H_{22}^c H_{33}^c - (H_{22}^c)^2} * x_2^{(i)} - \frac{(H_{23}^c V_2 - H_{33}^c V_3)}{H_{22}^c H_{33}^c - (H_{22}^c)^2} * x_3^{(i)} \right] \tag{6}$$

   The analysis will assume the wing is at zero angle of attack, and the concentrated force V_3 models the lift and is applied in the positive x_3 direction and at 1/4 chord length away from the leading edge. Drag is assumed to be negligible. V_3 = 256,000 N

   Therefore, Equation (6) can be reduced to:

$$\triangle f^{(i)} = -E_{(i)} A_{(i)} \left[\frac{-H_{23}^c V_3}{H_{22}^c H_{33}^c - (H_{22}^c)^2} * x_2^{(i)} + \frac{-H_{33}^c V_3}{H_{22}^c H_{33}^c - (H_{22}^c)^2} * x_3^{(i)} \right] \tag{7}$$

   For each segment, the shear flow can be calculated using:

$$f_{ij}^o = f_{i-1,j-1}^o + \triangle f^{(i)} \tag{8}$$

   Since the cut is made in the cross-section between the stringers 1 and 2, $f_{12}^o = 0$.

   Now, we start from the segment $\overline{12}$, all the way to $\overline{61}$ in counterclockwise.

Segment	Open Shear Flow
f^o_{12}	0 N/m
f^o_{23}	-5.1596 * 10^4 N/m
f^o_{34}	-1.1332 * 10^5 N/m
f^o_{45}	-9.3453 * 10^4 N/m
f^o_{56}	-4.7283 * 10^4 N/m
f^o_{61}	-9.1599 * 10^3 N/m

Table 3: Open Shear Flow in each Skin Segment

Calculations are down below:

$$\Delta f^{(i)} = -E_{(i)} A_{(i)} \cdot \left[\frac{-H^c_{23} \cdot V_3}{H^c_{22} H^c_{33} - (H^c_{23})^2} \cdot x_2^{(i)} + \frac{H^c_{33} \cdot V_3}{H^c_{22} H^c_{33} - (H^c_{23})^2} \cdot x_3^{(i)} \right]$$

$$= -E \cdot A \cdot V_3 \cdot \frac{1}{H^c_{22} H^c_{33} - (H^c_{23})^2} \left(-H^c_{23} \cdot x_2^{(i)} + H^c_{33} \cdot x_3^{(i)} \right)$$

$$= \frac{-72.39498 \cdot 10^9 \frac{N}{m^2} \cdot 0.00015 m^2 \cdot 256000 N \cdot 1.95 m}{6.8050 \cdot 10^7 \cdot 1.79545 \cdot 10^9 - (-2.994 \cdot 10^7)^2} \left(2.994 \cdot 10^7 \cdot x_2^{(i)} + 1.79545 \cdot 10^9 \, x_3^{(i)} \right)$$

$$= -4.4696 \cdot 10^{-5} \cdot \left(2.994 \cdot 10^7 \cdot x_2^{(i)} + 1.79545 \, x_3^{(i)} \right)$$

$f^o_{12} = 0$ due to the cut

$$f^o_{23} = f^o_{12} + \Delta f^{(2)} = 0 - 4.4696 \cdot 10^{-5} \cdot \left(2.994 \cdot 10^7 \cdot \underset{(2)}{\left(-\tfrac{11}{9} + 1\right)} + 1.79545 \cdot 10^9 \cdot \underset{(2)}{\left(\tfrac{6}{7} - \tfrac{305}{1449} \right)} \right)$$

$$= -51596.4133$$

$$f^o_{34} = f^o_{23} + \Delta f^{(3)} = -51596.4133 - 4.4696 \cdot 10^{-5} \left(2.994 \cdot 10^7 \cdot \underset{(3)}{\left(-\tfrac{11}{9}\right)} + 1.79545 \cdot 10^9 \underset{(3)}{\left(\tfrac{1144}{1449} \right)} \right)$$

$$= -113318.885$$

$$f^o_{45} = f^o_{34} + \Delta f^{(4)} = -113318.885 - 4.4696 \cdot 10^{-5} \left(2.994 \cdot 10^7 \cdot \underset{(4)}{\left(-\tfrac{20}{9}\right)} + 1.79545 \cdot 10^9 \underset{(4)}{\left(-\tfrac{305}{1449} \right)} \right)$$

$$= -93453.3066$$

$$f^o_{56} = f^o_{45} + \Delta f^{(5)} = -93453.3066 - 4.4696 \cdot 10^{-5} \left(2.994 \cdot 10^7 \cdot \underset{(5)}{\left(-\tfrac{2}{3} - \tfrac{11}{9}\right)} + 1.79545 \cdot 10^9 \underset{(5)}{\left(-\tfrac{1}{3} - \tfrac{305}{1449} \right)} \right)$$

$$= -47283.848$$

$$f^o_{61} = f^o_{56} + \Delta f^{(6)} = -47283.848 - 4.4696 \cdot 10^{-5} \left(2.994 \cdot 10^7 \left(-\tfrac{11}{9} + 1\right) + 1.79545 \cdot 10^9 \left(-\tfrac{6}{23} - \tfrac{305}{1449} \right) \right)$$

$$= -9159.93760$$

4. Calculate the closing shear flow, f_c, due to the shear load in each segment of the skin.

   Now, we have to close the cut and calculate the shear center based on f_o.

$$f_c = \frac{-\int_c \frac{f_o}{Gt} ds}{\int_c \frac{1}{Gt} ds} \tag{9}$$

   The shear modulus G is not provided in Figure 2, but this material is **isotropic**, therefore the shear modulus can be calculated by:

$$G = \frac{E}{2(1+v)} \tag{10}$$

   where v = Poisson's ratio = 0.33 (2), $t = 2.95\ mm$ (8)

   Therefore, G is constant. Now, we can reduce Equation 9 to:

$$f_c = \frac{-\int_c f_o ds}{\int_c ds} \tag{11}$$

   where $\int_c ds$ is the perimeter of the entire wing. The numerator $-\int_c f_o ds$ can be also expressed as:

$$-\int_c f_o ds = -\Sigma f_{oij} l_{ij} \tag{12}$$

   where l_{ij} is a length of each segment.

Segment	Length
l_{12}	11.8188 m
l_{23}	1.9698 m
l_{34}	3.0631 m
l_{45}	1.02102 m
l_{56}	3.25307 m
l_{61}	11.7111 m

Table 4: Length of each segment

   Putting in the values from Table 3 and Table 4 to Equation 12, we now have

$$f_c = \mathbf{24522.6884}\ N/m.$$

   Calculations are below:

$$f_c = \frac{1}{\text{perimeter}} \cdot -\int_c f_o \, ds$$

$$\left.\begin{array}{l} \overset{\frown}{34} = \frac{2\pi R}{4} = \frac{\pi R}{2} \\ \overset{\frown}{45} = \frac{2\pi \frac{R}{3}}{4} = \frac{\pi R}{6} \\ \overline{13} = \sqrt{R^2 + (7R)^2} = 7.07107 R \\ \overline{15} = \sqrt{\left(\frac{R}{3}\right)^2 + \left(7R + \frac{2R}{3}\right)^2} \end{array}\right\} \begin{array}{l} = 16.83937 R \\ = 32.8368 \, m \end{array}$$

$$= 7.6739 R$$

$$\overline{61} : \quad 7R + \frac{2}{3}R : 6R = 7.6739R : l$$

$$l = 6.0057 R$$

$$\overline{56} : \quad \overline{15} - \overline{61} = 1.66824 R$$

$$\overline{12} : \quad 7R : 6R = 7.07107R : l$$

$$l = 6.06092 R$$

$$\overline{23} : \quad \overline{13} - \overline{12} = 1.01015 R$$

$$\cancelto{0}{f_{12}^o} \cdot l_{12} + f_{23}^o \cdot l_{23} + f_{34}^o \cdot l_{34} + f_{45}^o \cdot l_{45} + f_{56}^o \cdot l_{56} + f_{61}^o \cdot l_{61}$$

$$= -51596.4133 \cdot 1.9698 - 113318.885 \cdot 3.0631 - 93453.3066 \cdot 1.02102$$
$$- 47283.848 \cdot 3.25307 - 9159.93760 \cdot 11.7111$$

$$= -805249.0833 \ N$$

$$f_c = \frac{-(-805249.0833)\,N}{32.8368 \, m}$$

$$= 24522.6884 \ \frac{N}{m}$$

5. Calculate the shear center, x_{2k}. Report its location relative to point O in Figure 2.

Using the property of the shear center, we use the point O for convenience in calculating x_{2k}, where k is the center of symmetry.

$$x_{2k} V_3 = \int_c (f^o + f_c) r_o ds = \Sigma 2 f_{oij} A_{ij} + f_c * 2A_{tot} \tag{13}$$

As V_3 is given, we can calculate x_{2k} by summing up each segment. The area of each segment is defined as an area bounded by $\overline{ij}$ and the point O.

Segment	Area
A_{o12}	11.408 m^2
A_{o23}	1.9072 m^2
A_{o34}	2.987 m^2
A_{o45}	0.754 m^2
A_{o56}	0.964 m^2
A_{o61}	3.472 m^2

Table 5: Area of each segment

Using Equation 13, we obtain:

$$x_{2k} = \textbf{-0.4495} \ m\textbf{.}$$

Calculations are below:

$$V_3 x_{2k} = \int_c (f^\circ + f_{closing}) r_o \, ds$$

$$= 2A_{tot} f_c + \int f^\circ r_o \, ds$$

$$A_{O23} = \frac{1}{2} R \cdot \frac{R}{7} + \frac{1}{2} R \cdot \frac{6R}{7} = 1.9012 \ m^2$$

$$A_{O12} = \frac{1}{2} \cdot 7R \cdot R - A_{O23} = 11.408 \ m^2$$

$$A_{O34} = \frac{1}{4} \pi R^2 = 2.987 \ m^2$$

$$A_{O45} = \frac{1}{4} \pi \left(\frac{R}{3}\right)^2 + \frac{1}{2}\left(\frac{R}{3}\right)\left(\frac{2R}{3}\right) = 0.754 \ m^2$$

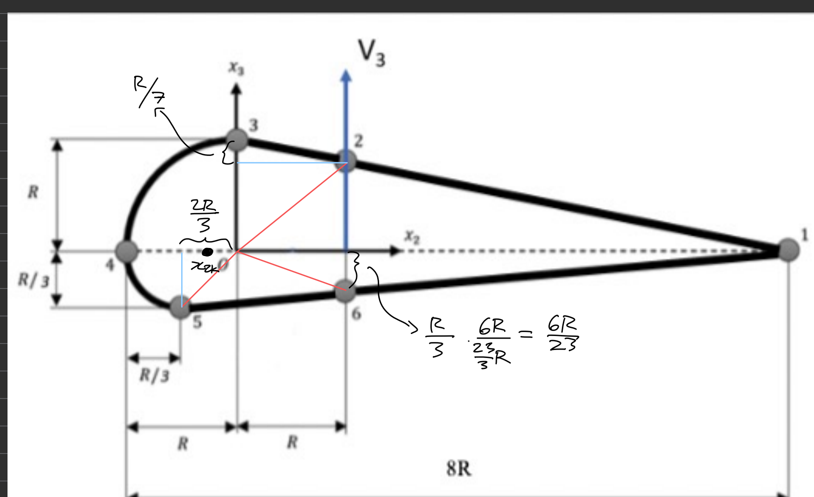
$$A_{O61} = \frac{1}{2} R \cdot \frac{6R}{23} + \frac{1}{2} \cdot 6R \cdot \frac{6R}{23} = 3.472 \ m^2$$

$$A_{O56} = \frac{1}{2} \cdot \frac{R}{3} \cdot \frac{23}{3} R - \frac{1}{2} \cdot \frac{R}{3} \cdot \frac{2R}{3} - A_{O61} = 0.964 \ m^2$$

$$\Rightarrow A_{tot} = 21.486 \ m^2$$

$$x_{2k} = \frac{2}{V_3}\left(A_{tot} f_c + A_{O12} f_{12}^\circ + A_{O23} f_{23}^\circ + A_{O34} f_{34}^\circ + A_{O45} f_{45}^\circ + A_{O56} f_{56}^\circ + A_{O61} f_{61}^\circ\right)$$

$$= -0.4495 \ m \quad (0.4495 \ m \text{ to the left of } O)$$



6. Calculate the torsion shear flow, f_t, due to the torque between the shear center location and the line of application of the applied force V_3.

   As k is to the left of the point O and the point where V_3 is applied, the torsional moment M_t is positive due to the sign convention. (counterclockwise is positive) Therefore, we use:

$$M_t = V_3 * |x_{2k} + R| \tag{14}$$

$$M_t = \mathbf{6.1426} * 10^5\ Nm.$$

Torsion shear flow, f_t, can be obtained using the Bredt-Batho formula:

$$f_t = \frac{M_t}{2A_{tot}} \tag{15}$$

Summing up all the areas of Table 5, we get $A_{tot} = 21.486\ m^2$. With A_{tot} and Equation 15, we get the torsional shear flow:

$$f_t = \mathbf{1.4295} * 10^5\ N/m.$$

7. Calculate the total shear flow, f_{ij}^{total}, and the shear stress in **each** skin segment. *Tabulate the values in your report.*

   Finally, the total shear flow, f_{ij}^{total}, can be obtained by:

$$f_{ij}^{total} = f_{oij} + f_c + f_t \tag{16}$$

where both f_c and f_t are constant.

In order to calculate shear stress, we need to look at the equilibrium equation:

$$\frac{\delta n}{\delta x_1} + \frac{\delta f}{\delta s} = 0 \tag{17}$$

In the stringer-skin model, we do not consider the area of the skin. Therefore, the skin does not carry axial stress. f = shear flow = $\tau_s * t(s)$. The thickness of each skin segment is constant.

$$\tau_{sij} = \frac{f_{ij}}{t} \tag{18}$$

Calculations were performed in Matlab (4.3).

Skin Segment	Total Shear Flow	Shear Stress
f_{12}	$3.8815 * 10^4 N/m$	$13158 * 10^7 Pa$
f_{23}	$-1.2770 * 10^4 N/m$	$-4.3318 * 10^6 Pa$
f_{34}	$-7.4503 * 10^4 N/m$	$-2.5255 * 10^7 Pa$
f_{45}	$-5.4643 * 10^4 N/m$	$-1.8521 * 10^7 Pa$
f_{56}	$-8.4657 * 10^3 N/m$	$-2.8697 * 10^6 Pa$
f_{61}	$2.9657 * 10^4 N/m$	$1.0053 * 10^7 Pa$

Table 6: Total Shear Flow and Shear Stress in Each Skin Segment

2 Deliverable 2: Report

2.1 Discussion

1. Assuming a safety factor of 1.5 for ultimate load, will the section fail in shear in the given scenario? Tabulate answers in your report for each layer.

   Once the total shear flow is known on each skin segment, we can now assess whether there is failure, using a safety factor of 1.5, which is a typical value for aircraft structures.

Skin Segment	Safty Factor
FOS_{12}	18.9
FOS_{23}	57.3
FOS_{34}	9.8
FOS_{45}	13.4
FOS_{56}	86.5
FOS_{61}	24.7

   Table 7: Safety factor of each skin segment

   The Ultimate Shear Stress as found in MIL-HDBK-5J is 36 ksi, which is equivalent to $2.4822 * 10^8 Pa$. Using FOS $= \frac{F_{su}}{\tau_{ij}}$, all skin segments were found to be well above the desired safety factor of 1.5. Therefore, the section does not fail in shear in the given scenario.

2. Regardless of the failure analysis results, discuss how the section can be further strengthened against shear stress.

   Regardless of the results of failure analysis, the section can be further strengthened against shear stress by increasing the thickness of the leading edge. As shown in Table 2, the airfoil has cross-bending stiffness, making structural integrity critical due to the direct influence of aerodynamic forces on performance. From Table 7, segments 34 and 45 - the two frontal parts of the airfoil — experience significantly higher shear flow. Since shear stress is inversely proportional to material thickness and shear flow is assumed constant (17), increasing the thickness of the leading edge would improve the airfoil's resistance to aerodynamic forces and improve performance.

3 Deliverable 3: Recorded Presentation

https://video.ucdavis.edu/media/Sydney%2C+Recorded+Presentation+3/1_otcrkyvr

Contributions of each member:

- Deliverable 1: Kaitaro and Jackson worked together
- Deliverable 2: All three members worked together
- Deliverable 3: Sydney

4 Appendix

4.1 Property Table

MIL-HDBK-5J
31 January 2003

Table 3.2.3.0(e_4). Design Mechanical and Physical Properties of Clad 2024 Aluminum Alloy Sheet and Plate—Continued

Specification	AMS-QQ-A-250/5									
Form	Flat sheet and plate									
Temper	T81		T851[a]			T861[a]				
Thickness, in.	0.010-0.062	0.063-0.249	0.250-0.499		0.500-1.000[b]	0.020-0.062	0.063-0.249	0.250-0.499	0.500[b]	
Basis	S	S	A	B	S	S	S	S	S	
Mechanical Properties:										
F_{tu}, ksi:										
L	64	67	65	66	63	65	70	68	67	
LT	62	65	65	66	63	64	69	68	67	
F_{ty}, ksi:										
L	57	59	56	58	56	59	65	62	61	
LT	54	56	56	58	56	58	64	62	61	
F_{cy}, ksi:										
L	55	57	56	58	56	59	65	62	61	
LT	55	57	57	59	56	61	67	65	64	
F_{su}, ksi	38	39	37	37	36	36	39	39	38	
F_{bru}, ksi:										
(e/D = 1.5)	96	100	99	100	96	99	107	105	104	
(e/D = 2.0)	122	127	127	129	123	128	138	136	134	
F_{bry}, ksi:										
(e/D = 1.5)	78	83	83	86	83	84	93	90	88	
(e/D = 2.0)	90	94	98	101	98	99	109	105	104	
e, percent (S-basis):										
LT	5	5	5	...	5	3	4	4	4	
E, 10^3 ksi:										
Primary	10.5	10.5	10.7			10.5	10.5	10.5		
Secondary	9.5	10.0	10.2			9.5	10.0	10.2		
E_c, 10^3 ksi:										
Primary	10.7	10.7	10.9			10.7	10.7	10.9		
Secondary	9.7	10.2	10.4			9.7	10.2	10.4		
G, 10^3 ksi	...									
μ	0.33									
Physical Properties:										
ω, lb/in.^{3}	0.100									
C, K, and α	...									

a Bearing values are "dry pin" values per Section 1.4.7.1.
b These values have been adjusted to represent the average properties across the whole section, including the 2-½ percent nominal cladding thickness.

Figure 2: Table we used for material properties

4.2 Wing Geometry Parameters

Component	Dimensions
R	1.95 m
A_s	150 mm^2
t	2.95 mm
c	8R

Table 8: Wing Geometry Parameters of JetZero

4.3 MATLAB Code

Here is the Matlab code used for Question 5, 6, and 7 in Deliverable 1.

```
R = 1.95
t = 0.00295
V3 = 256000
fc = 24522.6884
f120 = 0
f230 = -5.1596 * 10^4
 f450 = -9.3453 * 10^4
f560 = -4.7283 * 10^4
f610 = -9.1599 * 10^3



A023 = 0.5*R*R/7 + 0.5*R*6*R/7;
A012 = 0.5*7*R^2 - A023
A023
A034 = 0.25*pi*R^2
A045 = 0.25*pi*(R/3)^2 + 0.5 *(R/3)*(2*R/3)
A061 = 0.5*R*6*R/23 + 0.5*6*R * 6*R/23
A056 = 0.5*R/3*23*R/3 - 0.5*R/3*2*R/3 - A061


Atot = A023+A034 + A045 + A056 + A061 + A012

x2k = (2*Atot*fc + 2*A023*f230 + 2*A034*f340 + 2*A045*f450 + 2*A056*f560 + 2*A061*f610)/V3

Mt = V3 * (abs(x2k) + R)

fsc = Mt/(2*Atot)

f12tot = f120 + fc + fsc
f23tot = f230 + fc + fsc
f34tot = f340 + fc + fsc
f45tot = f450 + fc + fsc
f56tot = f560 + fc + fsc
f61tot = f610 + fc + fsc

```

```
tau12 = f12tot/t
tau23 = f23tot/t
tau34 = f34tot/t
tau45 = f45tot/t
tau56 = f56tot/t
tau61 = f61tot/t


% FOS
Fsu = 36 %ksi
Fsu = Fsu * 6.895*10^6

FOS12 = Fsu/abs(tau12)
FOS23 = Fsu/abs(tau23)
FOS34 = Fsu/abs(tau34)
FOS45 = Fsu/abs(tau45)
FOS56 = Fsu/abs(tau56)
FOS61 = Fsu/abs(tau61)
```

Listing 1: Matlab Code for Deliverable 1